On the Transmission Rate Strategies in Cognitive Radios

WoNeCa-3

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Outline

Motivation

Problem formulation

Existing and proposed transmission rate models

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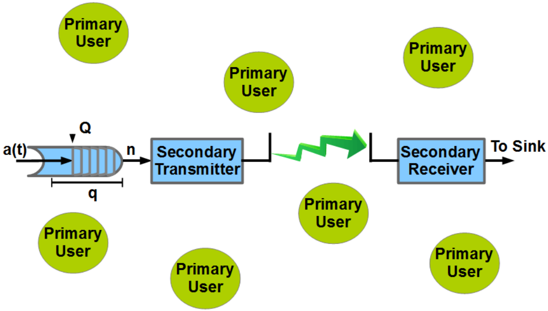
Motivation

- In cognitive radios, channel sensing errors have been considered generally in protecting primary users and maximizing transmission throughput
- Transmission rate strategies have not been investigated from a Data-link layer perspective

- Along with existing rate strategy, we proposed two other strategies
- We obtained effective capacity to understand the tradeoff between delay and rate strategies
- We performed low/high signal-to-noise ratio analysis
- There is not a unique strategy that is the best
- In IEEE Trans. Wireless Commun., Mar. 2016

Introduction

Cognitive radio channel model



Channel state

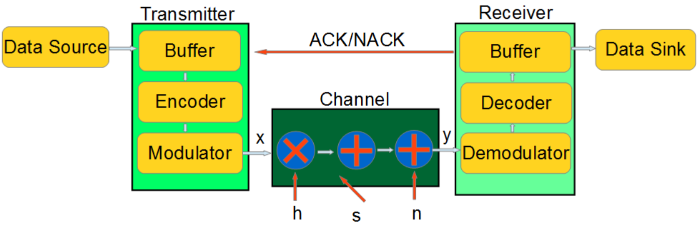
$\mathcal{H}_b$: Channel is busy
$\mathcal{H}_i$: Channel is idle

Channel sensing

$\widehat{\mathcal{H}}_b$: Sensed as busy
$\widehat{\mathcal{H}}_i$: Sensed as idle

Introduction

Input-output channel model



Transmission power

$$\widehat{\mathcal{H}}_b : P_b \leq P_{\max}$$
$$\widehat{\mathcal{H}}_i : P_i \leq P_{\max}$$

$P_b = \mu P_i$, $0 \leq \mu \leq 1$

Transmission rate

$$\widehat{\mathcal{H}}_b : R_b < C$$
$$\widehat{\mathcal{H}}_i : R_i < C$$

C : Channel Capacity

Problem Formulation

Channel sensing with errors

Channel is actually busy

Case 1 : Detected as busy

Case 2 : Detected as idle

Channel is actually idle

Case 3 : Detected as busy

Case 4 : Detected as idle

Sensing performance measures

Probability of detection

$$p_d = \frac{\Pr\{\text{Case 1}\}}{\Pr\{\text{Case 1} \cup \text{Case 2}\}}$$

Probability of false alarm

$$p_f = \frac{\Pr\{\text{Case 3}\}}{\Pr\{\text{Case 3} \cup \text{Case 4}\}}$$

$\widehat{\mathcal{H}}_b : R_b = ?$ and $\widehat{\mathcal{H}}_i : R_i = ?$

Problem formulation

$R_{b,i}$ may be set to the channel capacity because the channel fading, h, is known by the transmitter as well

Busy sensing

$$C_1 = f\left(\widehat{\mathcal{H}}_b, P_b, \mathcal{H}_b\right)$$

$$C_3 = f\left(\widehat{\mathcal{H}}_b, P_b, \mathcal{H}_i\right)$$

Idle sensing

$$C_2 = f\left(\widehat{\mathcal{H}}_i, P_i, \mathcal{H}_b\right)$$

$$C_4 = f\left(\widehat{\mathcal{H}}_i, P_i, \mathcal{H}_i\right)$$

Busy sensing

$R_b = C_1$ or $R_b = C_3$?
Given $C_1 \leq C_3$

Idle sensing

$R_i = C_2$ or $R_i = C_4$?
Given $C_2 \leq C_4$

Problem formulation

Example 1

1. Channel is sensed as busy and we set $R_b = C_1$
2. In *Case 1*, $R_b = C_1$ and R_b bits can be served
3. In *Case 3*, $R_b \leq C_3$ and R_b bits can be served
4. Due to false alarm, a chance of using a free channel is wasted by sending data at a lower rate

Example 2

1. Channel is sensed as idle and we set $R_i = C_4$
2. In *Case 4*, $R_i = C_4$ and R_i bits can be served
3. In *Case 2*, $R_i \geq C_2$ and 0 bits is possibly served
4. Due to miss-detection and interference from primary users, a transmission outage occurs

Existing and proposed transmission models

Optimistic policy (existing)

- Busy sensing : $R_b = C_1 \Longleftrightarrow$ Idle sensing : $R_i = C_4$
- In Cases 1 and 3, R_b bits are served
- In Cases 2 and 4, 0 and R_i bits are served, respectively

Conservative policy (proposed)

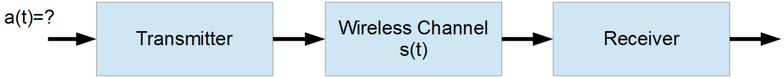
- Busy sensing : $R_b = C_1 \Longleftrightarrow$ Idle sensing : $R_i = C_2$
- In Cases 1 and 3, R_b bits are served
- In Cases 2 and 4, R_i bits are served

Greedy policy (proposed)

- Busy sensing : $R_b = C_3 \Longleftrightarrow$ Idle sensing : $R_i = C_4$
- In Cases 1 and 2, 0 bits are served
- In Cases 3 and 4, R_b and R_i bits are served, respectively

Effective capacity

System with a known service process s(t)



- $s(t) \in \{0, R_b, R_i\}$ in our model

Effective capacity

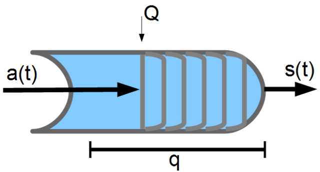
Dual of effective bandwidth; maximum constant arrival rate a stochastic service process can sustain under certain QoS constraints specified by θ

For a stable system, a(t)=?

$$C_E(\theta) = -\lim_{t \to \infty} \frac{1}{t\theta} \log_e \mathbb{E}\left\{e^{-\theta \sum_{\tau=1}^{t} s(\tau)}\right\}$$

What to infer from θ?

Queue in steady-state



$$\theta = -\lim_{q\to\infty} \frac{\log \Pr\{Q > q\}}{q}$$

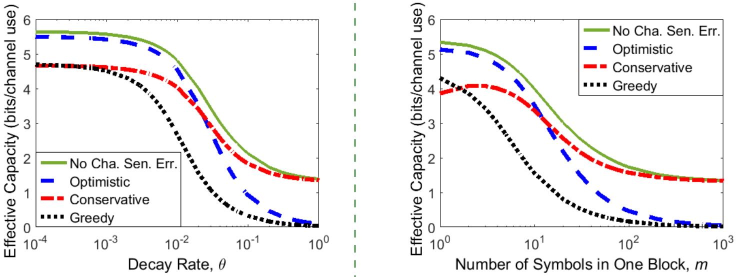
- For large q: $\Pr\{Q > q\} \approx e^{-\theta q}$
- Larger $\theta \to$ stricter constraints on buffer
- Smaller $\theta \to$ looser constraints on buffer

Properties of effective capacity

1. $\lim_{\theta\to\infty} C_E(\theta) \implies$ minimum service rate
2. $\lim_{\theta\to 0} C_E(\theta) \implies$ average service rate

Numerical results

Effective capacity vs. decay rate and symbol block size



- $P_{int} = 20$ dB and $P_{max} = 20$ dB
- m=50 (left figure), $\theta = 0.1$ (right figure)
- $p_d = 0.95$, $p_f = 0.1$ and interference = 5 dB

Low/high signal-to-noise ratio regime

Notes

$C_E(\theta, \gamma)$ is concave in the space defined by signal-to-noise ratio (γ)

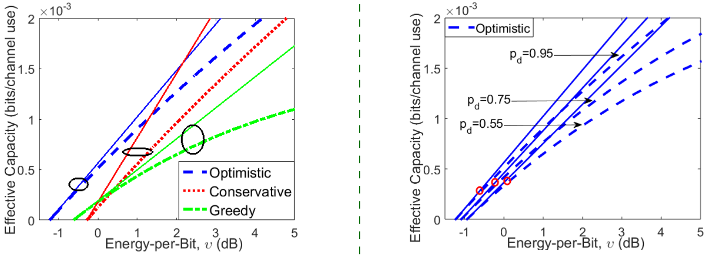
Low signal-to-noise ratio

- Energy-per-bit : $\upsilon = \frac{\gamma}{C_E(\theta,\gamma)}$
- υ_{min}: The minimum energy-per-bit is obtained as signal-to-noise ratio goes to zero, i.e., $\gamma \to 0$
- $\mathcal{S}_0$: Minimum υ and slope of the effective capacity versus υ (in dB) curve at υ_{min}

High signal-to-noise ratio

- $\mathcal{S}_\infty = \lim_{\gamma \to \infty} \frac{C_E(\theta,\gamma)}{\log_2 \gamma}$: High signal-to-noise ratio slope in bits/channel use (3 dB)
- $\mathcal{L}_\infty$: Power offset with respect to a reference channel having the same slope

Effective capacity vs. energy-per-bit υ

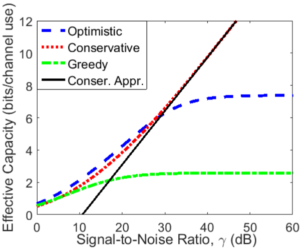


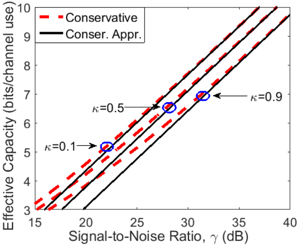
Solid lines are low signal-to-noise ratio approximations of the corresponding effective capacities

- m=200 and $\theta = 5$

Numerical results

Effective capacity vs. energy-per-bit υ





Solid lines are high signal-to-noise ratio approximations of the effective capacity in Conservative policy

- $\kappa = \frac{\theta m}{\log_e 2}$
- $\kappa = 0.9$ (left figure)

Thank You

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Questions

Backup — Effective capacity

Effective capacity as a function of signal-to-noise ratio (γ) and decay rate (θ)

$$C_E(\theta, \gamma) = \max_{p_d P_b + (1-p_d) P_i \leq P_{int}} -\frac{1}{m\theta} \log_e \frac{1}{2} \mathbb{E}_h \left\{ A + \sqrt{B^2 + 4C} \right\}$$

$$A = p_{b1} e^{-\theta R_1} + p_{b2} e^{-\theta R_2} + p_{i3} e^{-\theta R_3} + p_{i4} e^{-\theta R_4},$$

$$B = p_{b1} e^{-\theta R_1} + p_{b2} e^{-\theta R_2} - p_{i3} e^{-\theta R_3} - p_{i4} e^{-\theta R_4},$$

$$C = (p_{b3} e^{-\theta R_3} + p_{b4} e^{-\theta R_4})(p_{i1} e^{-\theta R_1} + p_{i2} e^{-\theta R_2})$$

p_{bk} and p_{ik} are functions of α, β, p_d, p_f for $k \in \{1, 2, 3, 4\}$

Optimistic policy : $R_1 = R_3 = C_1, R_2 = 0$ and $R_4 = C_4$

Conservative policy : $R_1 = R_3 = C_1$ and $R_2 = R_4 = C_2$

Greedy policy : $R_1 = R_2 = 0, R_3 = C_3$ and $R_4 = C_4$

Low/high signal-to-noise ratio regime

Remarks

- υ_{min} does not depend on θ in all transmission models
- $\mathcal{S}_0$ is a function of θ in all policies
- υ_{min} and $\mathcal{S}_0$ do not depend on the state transition probabilities of primary users in Conservative policy
- υ_{min} and $\mathcal{S}_0$ do not depend on the correlation dynamics of primary users' sampled signals in Greedy policy
- υ_{min} and $\mathcal{S}_0$ depend on p_d and p_f only in Optimistic policy

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- $\mathcal{S}_\infty = 0$ in Optimistic and Greedy policies
- $\mathcal{S}_\infty = 1$ if $\frac{\theta m}{\log_e 2} = \kappa < 1$, and $\mathcal{S}_\infty = \frac{1}{\kappa}$ otherwise in Conservative policy

High signal-to-noise ratio regime

High signal-to-noise ratio regime can be considered when:

1) There is no strict interference power constraint
2) Secondary users internal power limits are very high

Define

$\mathcal{S}_\infty = \lim_{\gamma \to \infty} \frac{C_E(\theta,\gamma)}{\log_2 \gamma}$: High signal-to-noise ratio slope in bits/channel use (3 dB)

$\mathcal{L}_\infty = \lim_{\gamma \to \infty} \left\{ \log_2 \gamma - \frac{C_E(\theta,\gamma)}{\mathcal{S}_\infty} \right\}$: Power offset with respect to a reference channel having the same slope

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Approximation : $C_E = \mathcal{S}_\infty \left[\log_2 \gamma - \mathcal{L}_\infty\right] - o(1)$

Remarks

- $\mathcal{S}_\infty = 0$ in Optimistic and Greedy policies
- $\mathcal{S}_\infty = 1$ if $\frac{\theta m}{\log_e 2} = \kappa < 1$, and $\mathcal{S}_\infty = \frac{1}{\kappa}$ otherwise in Conservative policy

Low signal-to-noise ratio regime

Notes

1) $C_E(\theta, \gamma)$ is concave in the space defined by γ
2) The minimum energy-per-bit is obtained as signal-to-noise ratio goes to zero, i.e., $\gamma \to 0$

Define

Energy-per-bit : $\upsilon = \frac{\gamma}{C_E(\theta,\gamma)}$

Minimum υ : $\upsilon_{\text{min}} = \lim_{\gamma \to 0} \frac{\gamma}{C_E(\theta,\gamma)} = \frac{1}{\dot{C}_E(\theta,0)}$

Slope of the effective capacity versus υ (in dB) curve at υ_{min} :

$$\mathcal{S}_0 = \lim_{\upsilon \downarrow \upsilon_{\text{min}}} \frac{C_E(\upsilon)}{10\log_{10}\upsilon - 10\log_{10}\upsilon_{\text{min}}} 10\log_{10} 2$$

$$= \frac{2(\dot{C}_E(\theta,0))^2}{-\ddot{C}_E(\theta,0)} \log_e 2 \text{ bits/channel use/(3 dB)}$$