Window Flow Control Systems with Random Service

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Joint work with Prof. Jörg Liebeherr and Prof. Almut Burchard

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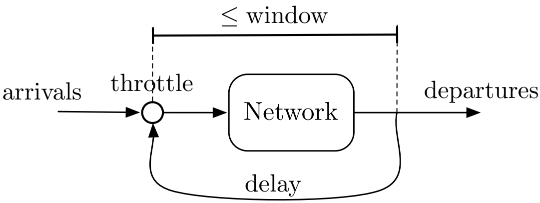
Content

Feedback system

- Feedback system:



- For the analysis we use network calculus methodology
- Network calculus has analyzed feedback systems under deterministic assumptions

Open problem in network calculus

Analysis of feedback systems with probabilistic assumptions

Related work

- Performance bonds for flow control protocols[1]
  - Deterministic analysis
  - Min-plus algebra
  - Window flow control model
- A min,+ system theory for constrained traffic regulation and dynamic service guarantees[2]
  - Deterministic analysis
  - Min-plus algebra
  - Window flow control model
- TCP is max-plus linear[3]
  - Deterministic service process
  - Max-plus algebra
  - TCP Tahoe and TCP Reno

[1] R. Agrawal et al. "Performance bonds for flow control protocols". In: *IEEE/ACM Transactions on Networking* 7.3 (1999), pp. 310–323.

[2] C.-S Chang et al. "A min,+ system theory for constrained traffic regulation and dynamic service guarantees". In: *IEEE/ACM Transactions on Networking* 10.6 (2002), pp. 805–817.

[3] F. Baccelli and D. Hong. "TCP is max-plus linear and what it tells us on its throughput". In: *ACM SIGCOMM* 30.4 (2000), pp. 219–230.

Related work

- TCP congestion avoidance[4]
  - Deterministic analysis
  - Min-plus algebra
  - Window flow control model
  - TCP Vegas and Fast TCP
- Window flow control in stochastic network calculus[5]
  - Stochastic analysis
  - Min-plus algebra
  - Window flow control model

[4] M. Chen et al. "TCP congestion avoidance: A network calculus interpretation and performance improvements". In: *IEEE INFOCOM*. vol. 2. 2005, pp. 914–925.

[5] M. Beck and J. Schmitt. "Window flow control in stochastic network calculus - The general service case". In: *ACM VALUETOOLS*. Jan. 2016.

Bivariate network calculus

$$(f \wedge g)(s,t) = \min\{f(s,t), g(s,t)\}$$
$$(f \otimes g)(s,t) = \min_{s \le \tau \le t}\{f(s,\tau) + g(\tau,t)\}$$
$$(f \otimes g)(s,t) \neq (g \otimes f)(s,t)$$

- ($\wedge$,$\otimes$) operations form a non-commutative dioid over non-negative non-decreasing bivariate functions
- discrete-time domain $(t = 0, 1, 2, ...)$
- Sub-additive closure:

$$f^* \triangleq \delta \wedge f \wedge f^{(2)} \wedge f^{(3)} \wedge \ldots = \bigwedge_{n=0}^{\infty} f^{(n)}$$

where $f^{(n+1)} = f^{(n)} \otimes f$ for $n \geq 1$, $f^{(0)} = \delta$, and $f^{(1)} = f$

Moment-generating function network calculus[6]

- Moment-generating function of a random variable X:

$$M_X(\theta) = E\left[e^{\theta X}\right]$$

- Moment-generating function of operations $\otimes$ and $\oslash$:

$$M_{f\otimes g}(-\theta, s, t) \leq \sum_{\tau=s}^{t} M_f(-\theta, s, \tau) M_g(-\theta, \tau, t)$$

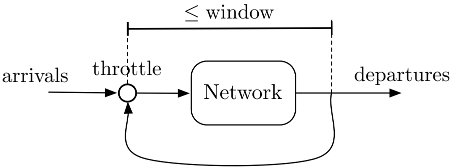
$$M_{f\oslash g}(\theta, s, t) \leq \sum_{\tau=0}^{s} M_f(\theta, \tau, t) M_g(-\theta, \tau, s)$$

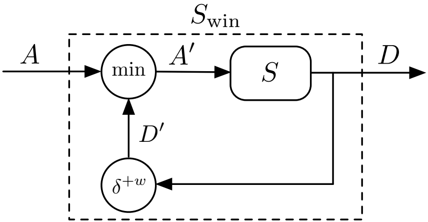
- For $Pr\Big(S(s,t) \leq \mathcal{S}^{\varepsilon}(s,t)\Big) \leq \varepsilon$, statistical service bound

$$\mathcal{S}^{\varepsilon}(s,t) = \max_{\theta>0} \frac{1}{\theta}\Big\{\log \varepsilon - \log M_S(-\theta, s, t)\Big\}$$

[6] M. Fidler. “An end-to-end probabilistic network calculus with moment generating functions”. In: *IEEE IWQoS*. 2006, pp. 261–270.

State-of-the-art: Window flow control





$$A' = \min\{A, D'\}$$

$$\delta^{+w}(s,t) = \begin{cases} w & s \geq t\,, \\ \infty & s < t \end{cases}$$

$$D' = D \otimes \delta^{+w} = D + w$$

$$\begin{aligned} A' - D &= \min\{A, D + w\} - D \\ &\leq D + w - D \\ &= w \end{aligned}$$

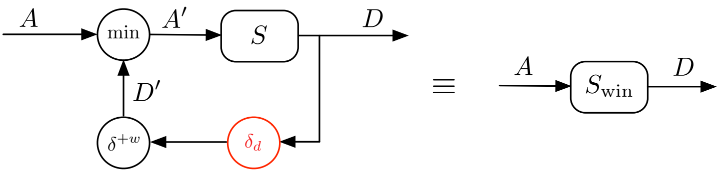
State-of-the-art: Window flow control

- Delay element represent feedback delay:

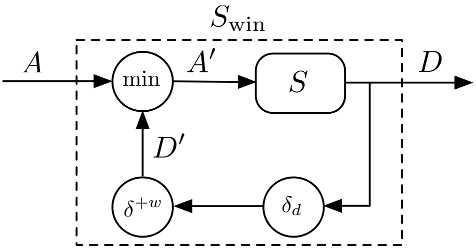
$$\delta_d(s,t) = \delta(s, t-d)$$

- Equivalent feedback service:

$$S_{\text{win}} = \left(S \otimes \delta_d \otimes \delta^{+w}\right)^* \otimes S$$



Results: Exact result



Feedback system with $w > 0$, $d \geq 0$ and with an **additive service process**

$$S(s, t) = \sum_{k=s}^{t-1} c_k$$

c_k's are arbitrary sequence of non-negative random variables

If feedback delay is one ($d = 1$),

$$S_{\text{win}}(s, t) = \sum_{k=s}^{t-1} \min\{c_k, w\}$$

Results: Upper and lower bounds

For the equivalent service process S_{win} of a general feedback system with window size $w > 0$, and feedback delay $d \geq 0$, we have

- Upper and lower bounds:

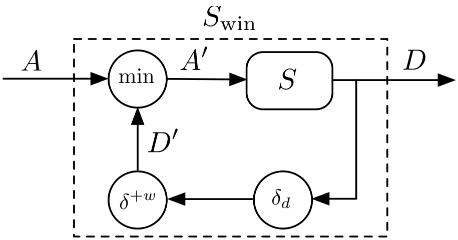
$$S'_{\text{win}}(s,t) < S_{\text{win}}(s,t) < \min\left\{S(s,t), \left\lceil \frac{t-s}{d} \right\rceil w\right\}$$

$S'_{\text{win}}(s,t)$ is the equivalent service process of the feedback system with window size $w' = w/d$ and feedback delay $d' = 1$

- The lower bound corresponds to the exact result

Results: Equivalent service

- Feedback system with window size $w > 0$ and delay $d \geq 0$:



$$S_{\text{win}}(s,t) = \bigwedge_{n=0}^{\lceil \frac{t-s}{d} \rceil} \left\{ \min_{C_n(s,t)} \left(\sum_{i=1}^{n} \Big(S(\tau_{i-1}, \tau_i - d) \Big) + S(\tau_n, t) \right) + nw \right\}$$

where $C_n(s,t)$ is given as

$$C_n(s,t) = \{ s = \tau_o \leq \cdots \leq \tau_n \leq t \,|\, \forall i = 0, \ldots, n \;\; \tau_i - \tau_{i-1} \geq d \}$$

Results: Feedback system with VBR

Variable Bit Rate (VBR) server

$$S(s, t) = \sum_{k=s}^{t-1} c_k$$

where c_k's are independent and identically distributed random variables

For a feedback system with VBR server with window size $w > 0$ and delay $d \geq 0$:

$$M_{S_{\text{win}}}(-\theta, s, t) \leq \left(M_c(-\theta)^d + de^{-\theta w}\right)^{\left\lfloor \frac{t-s}{d} \right\rfloor}$$

$M_c(\theta)$ is the moment-generating function of c_k,

$$M_c(\theta) = E\left[e^{\theta c_k}\right]$$

Results: Feedback system with MMOO

Markov-modulated On-Off (MMOO) server operates in two states:

- ON (state 1): The server transmits a constant amount of $P > 0$ units of traffic per time slot, $c_k = P$
- OFF (state 0): The server does not transmit, $c_k = 0$

The MMOO server offers an additive service process

$$S(s, t) = \sum_{k=s}^{t-1} c_k$$

For a feedback system with MMOO server with window size $w > 0$ and delay $d \geq 0$, if $p_{01} + p_{10} < 1$:

$$M_{S_{\text{win}}}(-\theta, s, t) \leq \left(m_+(-\theta)^d + de^{-\theta w} \right)^{\left\lfloor \frac{t-s}{d} \right\rfloor}$$

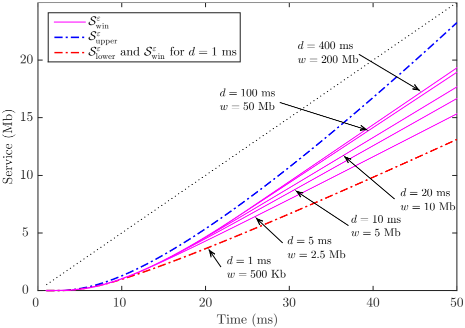
$m_+(\theta)$ is the larger eigenvalue of the matrix

$$L(\theta) = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{\theta P} \end{pmatrix}$$

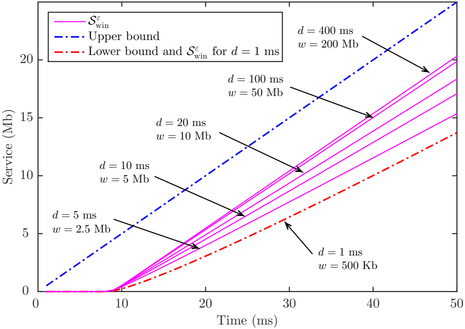
Numerical results: Statistical service bounds

$$\mathcal{S}^{\varepsilon}_{\text{win}}(s,t) = \max_{\theta>0} \frac{1}{\theta}\Big\{\log\varepsilon - \log M_{S_{\text{win}}}(-\theta,s,t)\Big\}$$

VBR server with exponential c_k



MMOO server with $p_{00} = 0.2$, $p_{11} = 0.9$, $P = 1.125$ Mb

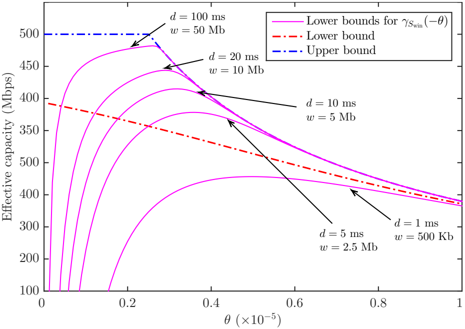


Average rate = 1 Gbps, time unit = 1 ms, $w/d = 500$ Mbps, $\varepsilon = 10^{-6}$

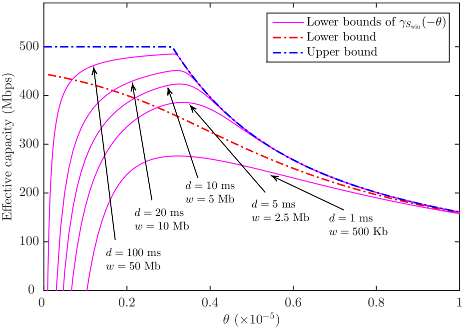
Numerical results: Effective capacity

$$\gamma_{S_{\text{win}}}(-\theta) = \lim_{t\to\infty} -\frac{1}{\theta t} \log M_{S_{\text{win}}}(-\theta, 0, t)$$

VBR server with exponential c_k



MMOO server with
$p_{00} = 0.2$, $p_{11} = 0.9$, $P = 1.125$ Mb

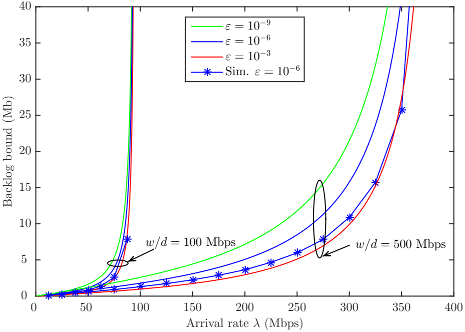


Average rate = 1 Gbps, $w/d = 500$ Mbps

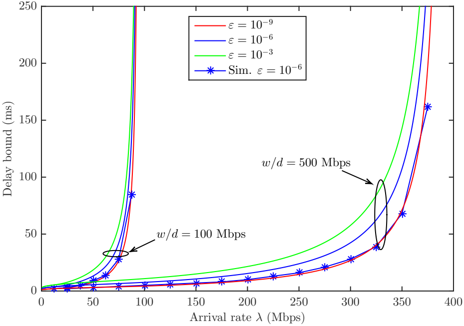
Numerical results: Backlog and delay bounds

$$A(s,t) = \sum_{k=s}^{t-1} a_k \quad \text{with exponential } a_k \text{ and average rate } \lambda$$

Backlog bound

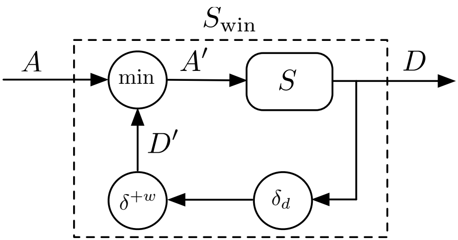


Delay bound



Exponential VBR, time unit $= 1$ ms, feedback delay $d = 1$ ms

Conclusions



Results:

- Exact results
- Upper and lower service bounds
- Equivalent service of the feedback system
- Bounds for a feedback system with VBR server
- Bounds for a feedback system with MMOO server
- Backlog and delay bounds

Technical Report - July 2016

- A. Shekaramiz, J. Liebeherr, and A. Burchard. Window Flow Control Systems with Random Service.
  arXiv:1507.04631, July 2015.

Thank you

Q & A