Transient Analysis for Wireless Networks

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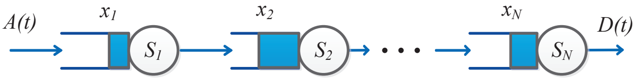
Joint work with James Gross and Hussein Al-Zubaidy

Network Model for Transient Analysis



- A finite sequence of time-critical data bits/packets arrive in $[t_0, t_0 + T]$, where $t_0 \geq 0$ and $T < \infty$
- x_n: backlog of wireless link n at t_0
- Service at each link is given by capacity of fading channel

What is the end-to-end delay of time-critical data?

Motivation

The network model is useful in studying emerging Machine Type Communication (MTC) or URLL applications in 5G

- Factory automation, motion control, intelligent transport systems, automated guided vehicles etc.
- Data is generated in short bursts
- For optimal control end-to-end delay may not exceed a few milliseconds

This necessitates the understanding of short-term behaviour of the wireless network

Challenges

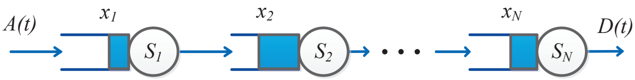
1. Effect of instantaneous backlog on the end-to-end delay has not been tackled
2. Non-stationary behaviour of multi-hop wireless networks is not well understood

Methodologies

1. *Queueing analysis* quickly becomes intractable for transient state system even for M/M/1 queue [Morse1958]
2. *Effective capacity* analysis can only be used for asymptotic measures
3. *Stochastic network calculus* can be used for transient analysis:
   - Many works studied stationary performance of wireless networks
   - Non-stationary service curves were developed to analyse temporary phases of communication networks [Becker2015]

We use stochastic network calculus to derive probabilistic delay bounds in transient state

Network Description: Arrival Process



- Fluid flow, discrete-time queuing model for N-hop wireless network starting at time $t_0 \geq 0$
- Consider (σ, ρ)-bounded arrival process $A(t) = A(t_0, t)$, where

$$A(u,t) = \sum_{i=u}^{t-1} a_i,\ t_0 \leq u \leq t$$

  - a_i is the arrival increment in slot i
  - $a_i = 0$ for all $i \notin [t_0, T)$

Service Process

- Rayleigh block fading wireless channels
- Cumulative service at n^{th} wireless link

$$S_n(u,t) = W \sum_{i=u}^{t-1} \log_2(1 + \gamma_n(i)).$$

  - W - bandwidth
  - $\gamma_n(i)$ - instantaneous SNR at link n during slot i
- Assume channels are i.i.d. both across links and time slots

Violation Probability

- **Objective:** Study delay violation probability $\mathbb{P}(W(t) > w)$
  - w: delivery deadline for the arrivals
  - End-to-end virtual delay:

$$W(t) = \inf\left\{ i \geq 0 : A(t) + \sum_{n=1}^{N} x_n \leq D(t+i) \right\}.$$

We focus on computing tight upper bounds for $\mathbb{P}(W(t) > w)$.

Modelling Initial Backlog

- In network calculus framework all initial backlog is assumed *zero*
- To fit into this framework we model x_n as cross traffic at t_0:

$$A_n^c(t) = \min(\kappa(t - t_0), x_n)$$

  - $\kappa(t)$: burst function

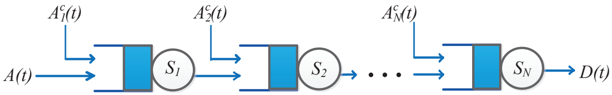


Figure: Equivalent model.

Our Work

1. Study off the shelf *Stationary* bound [Al-Zubaidy2016][1], by assuming arrivals happen over infinite time horizon
   - Bound is loose
   - Decay rate doesn't match simulation
2. Adapt the results of stationary bound and derive a transient bound - *State-Of-The-Art (SOTA) transient*
   - Tighter than stationary but still loose (two orders of magnitude)
   - Decay rate matches
3. Conduct independent analysis starting with basic principles of network calculus by incorporating initial backlogs - *Proposed transient* bound.
   - Around an order of magnitude.
   - Decay rate matches
   - Doesn't have closed-form expression

[1] Hussein Al-Zubaidy and Jorg Liebeherr and Almut Burchard,"Network-Layer Performance Analysis of Multihop Fading Channels", IEEE/ACM Transactions on Networking, vol. 24, no. 1, pp. 204–217, Feb 2016.

$(\min, \times)$ Network Calculus [Al-Zubaidy2016]

- Transform processes to *SNR domain* using exponential function
- $(\min, \times)$-convolution of $\mathcal{X}$ and $\mathcal{Y}$ is defined as

$$\mathcal{X} \otimes \mathcal{Y}(u,t) = \inf_{u \le v \le t} \{\mathcal{X}(u,v) \cdot \mathcal{Y}(v,t)\} \ .$$

Theorem 1

A probabilistic delay bound is given by $\mathbb{P}(\mathcal{W}(t) > w^{\varepsilon}) \le \varepsilon$, *where* w^{ε} *is the smallest* $w \ge 0$ *that satisfies* $\inf_{s>0} \{\mathcal{K}(s,\tau,t)\} \le \varepsilon$, *where* $\tau = t + w$ *and kernal*

$$\mathcal{K}(s,\tau,t) = \sum_{u=0}^{t} \mathcal{M}_{\mathcal{A}}(1+s,u,t)\mathcal{M}_{\mathcal{S}}(1-s,u,\tau) \, .$$

Mellin transform of $\mathcal{X}$, $\mathcal{M}_{\mathcal{X}}(s,u,t) = \mathcal{M}_{\mathcal{X}(u,t)}(s) = \mathbb{E}\left[\mathcal{X}^{s-1}(u,t)\right]$

Stationary Bound

- Mellin transform of the service increment:
  $\mathcal{M}_{\mathcal{S}}(1-s,u,\tau) = [V(s)]^{\tau-u}$
  - $V(s) = e^{\frac{1}{\bar{\gamma}}} \bar{\gamma}^{\frac{-sW}{\log 2}} \Gamma\left(1 - \frac{sW}{\log 2}, \bar{\gamma}^{-1}\right)$, $\bar{\gamma}$ is average SNR

Theorem 2

A probabilistic end-to-end delay bound for a cascade of N i.i.d. Rayleigh block fading channels with (σ, ρ)-bounded arrivals and (σ_c, ρ_c)-bounded cross traffic is given by

$$\mathbb{P}(\mathcal{W}(t) > w) \leq \inf_{s>0} \left\{ \frac{e^{s(-\rho w + \sigma + N\sigma_c)}}{(1 - V_0(s))^N} . \min\{1, (V_0(s))^w (w+1)^{N-1}\} \right\},$$

where $V_0(s) = e^{s(\rho+\rho_c)} V(s)$.

Performance of Stationary Bound

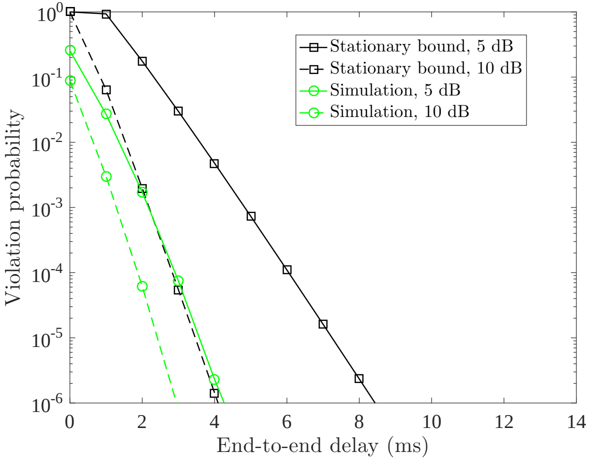


Figure: Violation probability vs end-to-end delay for a burst arrival $(T = 1)$ for a single link. $x_1 = 0$, $\rho = 20$, $\sigma = 0$.

SOTA Transient Bound

State-Of-The-Art (SOTA) transient bound is derived using Theorem 1.

- Tighter bounds for Mellin transforms of arrival and service processes
  - $\mathcal{M}_{\mathcal{A}}(1+s,u,t) \leq e^{s(\sigma \min\{1,t-u\}+\rho(t-u))}$
  - $\mathcal{M}_{\mathcal{S}}(1-s,u,\tau) \leq e^{sNx_{\mathsf{max}}}\binom{N-1+\tau-u}{\tau-u}(V_0(s)e^{-s\rho})^{\tau-u}$
- Substitute in Theorem 1:

$$\mathcal{K}(s,\tau,t) = e^{s(Nx_{\mathsf{max}}-\rho w)}\Big[\sum_{u=0}^{t-1} e^{s\sigma}\binom{N-1+\tau-u}{\tau-u}V_0^{\tau-u}(s) + \binom{N-1+w}{w}[V_0(s)]^w\Big]. \tag{1}$$

- Numerical analysis shows that this bound is loose

Proposed transient for Single-Hop

Theorem 3

For a single-hop scenario, an upper bound for $P\{\mathcal{W}(t) > w\}$ is given by

$$\min_{s>0}\left[\mathcal{A}^s(t)e^{sx_1}V^{\tau}(s) + \sum_{u=1}^{t-1}[\mathcal{A}(t)/\mathcal{A}(u)]^s V^{\tau-u}(s)\right].$$

Outline of proof:

- $\mathbb{P}\{\mathcal{W}(t) > w\} = P\{\mathcal{D}(\tau) < \mathcal{A}(t)e^{x_1}\}$
  - From network calculus $D(\tau) = \min_{0\le u\le\tau}[\mathcal{S}(\tau-u)\cdot\mathcal{A}(u)\cdot\mathcal{A}_1^c(u)]$
- Expand and use union bound and moment bound

Comparison of Upper Bounds for a Single-Hop

Recall that $V_0(s) = e^{(\rho+\rho_c)}V(s)$.

Stationary	$\inf_{s>0}\left\{\frac{e^{s(-\rho w+\sigma+x_1)}}{(1-V_0(s))}.\min\{1,(V_0(s))^w\}\right\}$
SOTA transient	$\min_{s>0}\left\{e^{s(x_1-\rho w)}(V_0(s))^w\left[e^{s\sigma}\cdot\frac{V_0(s)-(V_0(s))^{t+1}}{1-V_0(s)}+1\right]\right\}$
Proposed transient	$\min_{s>0}\left\{\mathcal{A}^s(t)e^{sx_1}V^\tau(s)+\sum_{u=1}^{t-1}[\mathcal{A}(t)/\mathcal{A}(u)]^s V^{\tau-u}(s)\right\}$

- Transient bounds are functions of t
- Proposed transient: initial backlog associated with only first term

Proposed transient for N-Hop Wireless Network

Theorem 4

For N-hop wireless network, an upper bound for $P\{\mathcal{W}(t) > w\}$ is given by

$$\min_{s>0} V^{\tau}(s)[\mathcal{A}(t)]^{s}\Big[\sum_{u=1}^{\tau}\sum_{u_1=1}^{u}\dots\sum_{u_{N-1}=1}^{\min(u_{N-2},t-1)}[\mathcal{A}(u_{N-1})]^{-s}V^{-u_{N-1}}(s) + \sum_{i=0}^{N-1}\binom{i+\tau-1}{\tau-1}e^{s\sum_{n=1}^{N-i}x_n}\Big]$$

- Time complexity - $O((N+\tau)!/N!\tau!)$, where $\tau = t + w$.

Performance comparison: Single Link

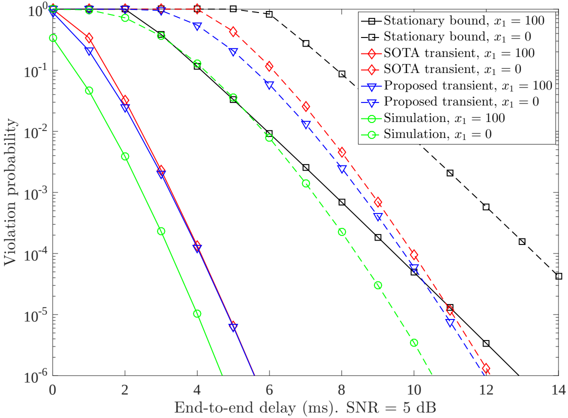


Figure: Burst arrival ($T = 1$) with different backlogs, $\rho = 0$ and $\sigma = 25$.

Performance comparison: Two-Hop Network

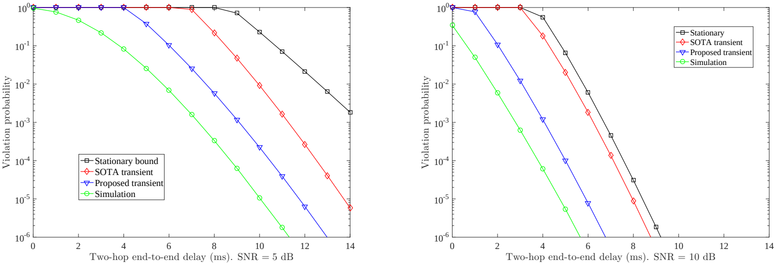


Figure: Two-hop network with $T = 5$, $x_n = 100, \rho = 25$ and $\sigma = 0$

Tightness of Proposed Transient: Three-Hop Network

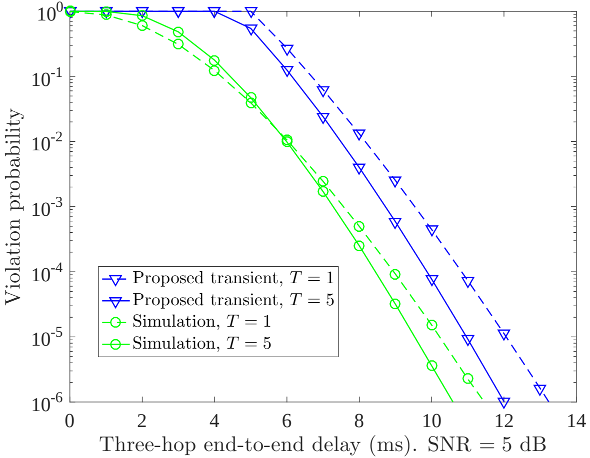


Figure: Three-hop network with $x_n = 33, \rho = 25$, $\sigma = 0$ and different T.

Summary

- Multi-hop wireless network with non-zero initial backlog at each hop
- Studied the end-to-end delay violation probability of a sequence of time-critical control packets
- Demonstrated the poor performance of the SOTA transient bound
- Derived new transient bound by using the first principles of network calculus and the state-of-the-art bounding techniques
- Decay rate of the proposed transient closely matches the decay rate from simulation and the gap is around an order of magnitude

Recent Work on Age of Information (AoI)

- AoI is a metric that measures the freshness/staleness of data received
  - Freshness of data is of prime importance in a network where source sends status updates to destination
- **Definition:** Time elapsed since the generation of the latest status update received at the destination

Our Work:[2]

- Analysed *age limit* violation probability using max-plus algebra
- Computed status-update rate that minimizes the violation probability

[2] with Hussein Al-zubaidy and James Gross "Statistical Guarantee Optimization for Age of Information for the D/G/1 Queue", to appear in AoI Workshop, IEEE INFOCOM, 2018.

Additional Slides

Related Work

Queueing theory:

- Works are sparse compared with stationary analysis
  - Transient state queueing analysis quickly becomes intractable (even for M/M/1 queue [Morse1958])
  - Approximations or numerical methods have been proposed
- Transient analysis for flows of shorter length for selection of TCP congestion window [Mellia2002]
  - Does not account for queuing affect along the route
- Transient analysis of ATM networks [Wang1996]
  - Limited by dependency on numerical methods

Related Work (contd.)

Effective Capacity:

- Devised to provide asymptotic delay and backlog performance
- Limited to stationary performance metrics

Stochastic Network Calculus:

- Several research works studied stationary performance of wireless networks
- Non-stationary service curves were developed to analyse temporary phases of communication networks [Becker2015]

Transient analysis of multi-hop wireless networks has not been attempted before

Numerical Results

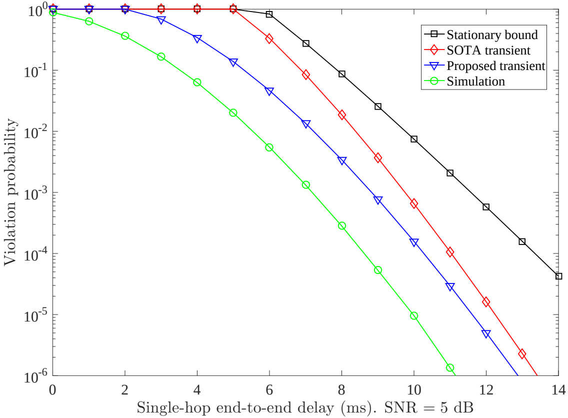
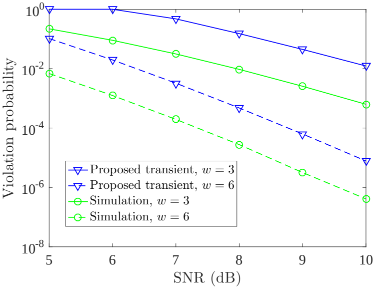


Figure: Arrival process with $T = 5$, $x_1 = 100, \rho = 25$ and $\sigma = 0$.

Tightness of Proposed Transient: Two-Hop Network



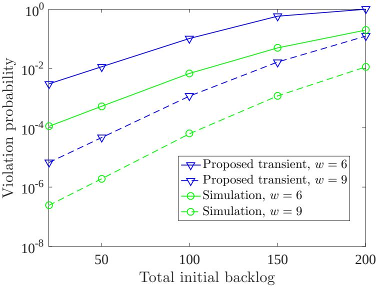


Figure: Two-hop network with $T = 5$, $\rho = 25$, $\sigma = 0$ and varying delay w.