A Statistical Property of Wireless Channel Capacity

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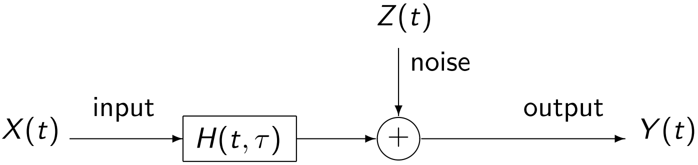
Motivation

- Wireless communication has been around for over a century, starting from Marconi's demonstration of wireless telegraphy around 1897.
- The mobile communication system renews every ten years, from 1G to 5G, through 1980s to 2020s.
- 5G will thrust mobile technology into the exclusive realm of general purpose technologies, like electricity and automobile.
- The promising technology for the future motivates to reflect on the fundamental theory.

Outline

1. What is the fundamental property of this stochastic process?
2. What is the hidden resource to be utilized in wireless channels?
3. What is the impact of self-interference on ad hoc network scalability?

Stochastic Channel Model



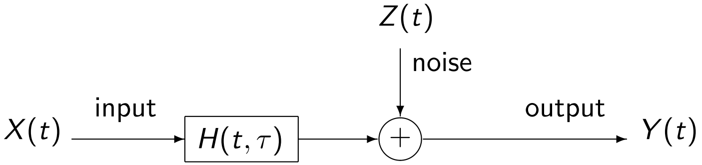
$$Y(t) = \int_{-\infty}^{\infty} H(t,\tau)X(t-\tau)d\tau + Z(t)$$

The wireless channel is modeled by a linear time-variant filter, with a baseband representation in discrete time and complex domain

$$y(t) = \sum_{\ell} h_{\ell}(t)x(t-\ell) + z(t),$$

where the filter taps are further modeled by random variables, e.g., in Rayleigh and Rice fading channels.

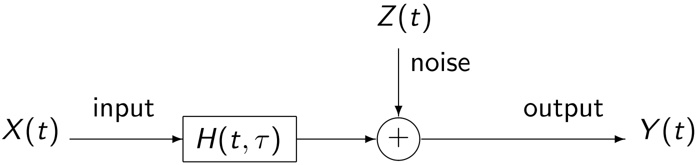
Channel Capacity



Conditional on a channel realization, the capacity is expressed as

$$C(t) = \max_{p(x)} I(X; Y | h(t)),$$

which is the maximum mutual information over input distribution, $p(x) = P\{X = x\}$.



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Considering the stochastic model of the filter tap, the instantaneous capacity is a random variable.

Light-tailed Behavior

For a flat fading channel with additive white Gaussian noise, the capacity is expressed as

$$C(t) = W \log_2 \left(1 + SNR|h(t)|^2\right),$$

which is the logarithm function of the fading process, thus

$$\overline{F}_C(x) = O(e^{-\theta x}) \text{ entails } \overline{F}_h(r) = O\left(r^{-\theta}\right).$$

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- If the distribution of the fading process is not heavier than fat tail, the distribution of the instantaneous capacity is light-tailed.
- If a wireless channel is Rayleigh, Rice, Nakagami-m, Weibull, or lognormal fading channel, its instantaneous capacity distribution is light-tailed.

A Fundamental Property

For a frequency-selective fading channel, the capacity is

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- It is a general property for a class of channel models.
- It holds for a series of fading scenarios.
- It assists in performance analysis.

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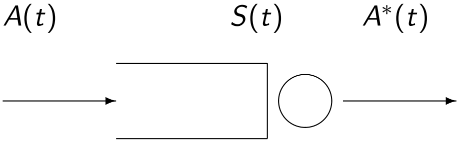
Queueing Principle



The queueing principle is expressed as the backlog recursion

$$B(t+1) = [B(t) + X(t)]^+ ,$$

where $X(t) = a(t) - C(t)$ is the instantaneous backlog increment. Assume $B(0) = 0$, the backlog is expressed as

$$B(t) = \sup_{0 \le s \le t} (A(s,t) - S(s,t)).$$

Performance Measure

For a lossless system, the output is the difference between the input and backlog,

$$A^*(t) = A(t) - B(t),$$

and the delay is defined via the input-output relationship, i.e.,

$$D(t) = \inf \{d \geq 0 : A(t - d) \leq A^*(t)\}.$$

It is the virtual delay that a hypothetical arrival has experienced on departure.

Dependence Classification

The capacity $\boldsymbol{C} = (C(1), \ldots, C(t))$ is said to have a positive dependence structure in the sense of supermodular order, if

$$\boldsymbol{C}_\perp \leq_{sm} \boldsymbol{C}_P,$$

or a negative dependence structure in the sense of supermodular order, if

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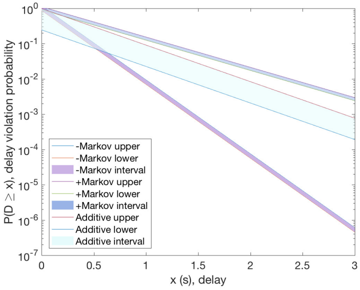
Intuitively, positive dependence implies that large or small values of random variables tend to occur together, while negative dependence implies that large values of one variable tend to occur together with small values of others.

Resource Identification

Consider a constant arrival

- Exponential bounding function for weak dependence
- Largest asymptotic decay rate for the same mean



$$\boldsymbol{C} \leq_{sm} \widetilde{\boldsymbol{C}},\ \forall t \in \mathbb{N} \implies S(t) \leq_{cx} \widetilde{S}(t),\ \forall t \in \mathbb{N} \implies \widetilde{\theta} \leq \theta$$

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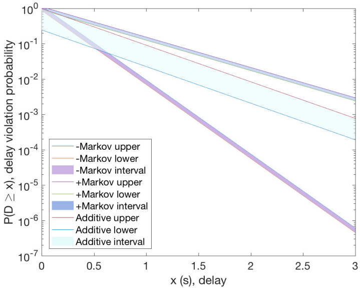

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In view of the delay gain, the negative dependence is identified as a hidden resource.

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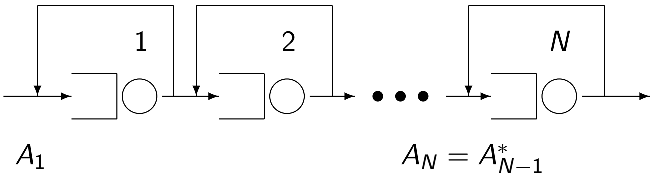
Queueing Network



- An instinct feature of the ad hoc wireless network is self-interference.
- A wireless channel with self-interference is a queueing system with feedback.

For a lossless system, the output is the difference between the input and backlog,

$$A^*(t) = A(t) - B(t),$$

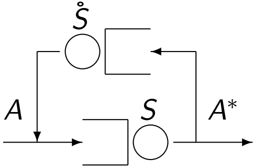
considering $B(t) = \sup_{0 \le s \le t}(A(s,t) - S(s,t))$, which is further represented by

$$A^*(t) = A \otimes S(t),$$

where $f \otimes g(s,t) = \inf_{s \le \tau \le t}\{f(s,\tau) + g(\tau,t)\}$ is the bivariate min-plus convolution. Since the operator is associative, the output of a N-hop network is expressed as

$$A_N^*(t) = A \otimes (S_1 \otimes \ldots \otimes S_N)(t).$$

Single-hop Analysis



$$\widetilde{A}(t) = A(t) + A^* \otimes \mathring{S}(t)$$

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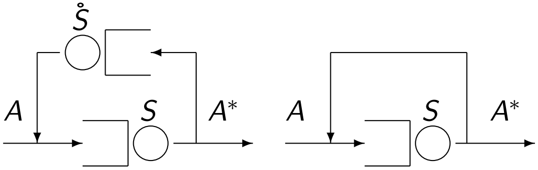

$\widetilde{A}(t) = A(t) + A^* \otimes \mathring{S}(t) \quad \widetilde{A}(t) = A(t) + A^*(t)$

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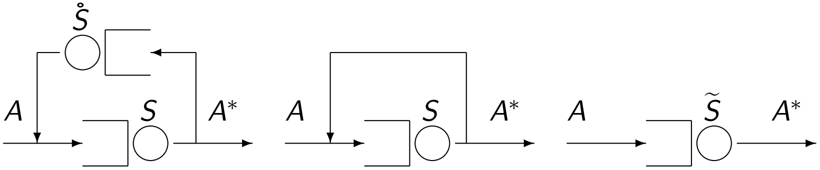

$\widetilde{A}(t) = A(t) + A^* \otimes \mathring{S}(t)$ $\quad$ $\widetilde{A}(t) = A(t) + A^*(t)$ $\quad$ $\widetilde{A}(t) = A(t)$

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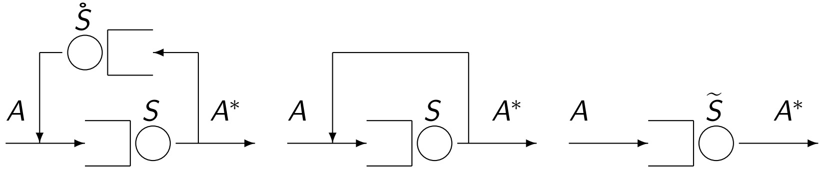

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$$\widetilde{S}(t) \;\geq\; S(t) - A^* \otimes \mathring{S}(t) \;\geq\; S(t) - A^*(t) \;\geq\; S(t) - A(t)$$

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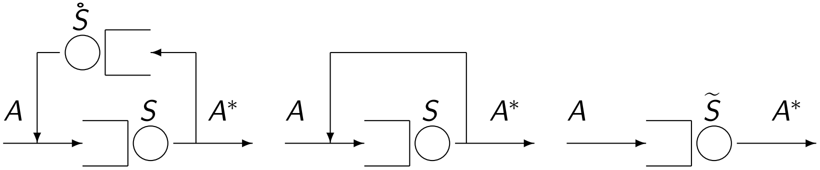

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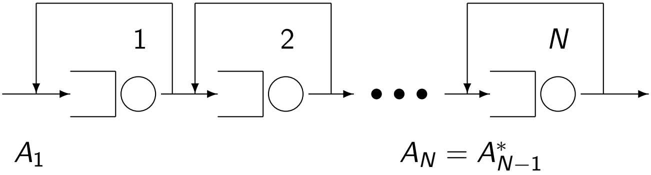
$$\widetilde{S}(t) \ \geq \ S(t) - A^* \otimes \mathring{S}(t) \ \geq \ S(t) - A^*(t) \ \geq \ S(t) - A(t)$$

A service lower bound is established with system causality

$$A^*(t) \leq A(t).$$

Multiple-hop Analysis



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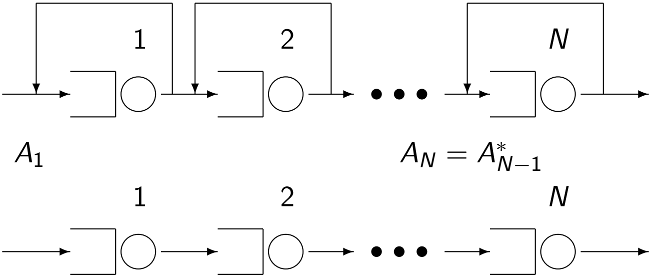

$(S_1 - A^*_1) \otimes \ldots \otimes (S_N - A^*_N)(t)$

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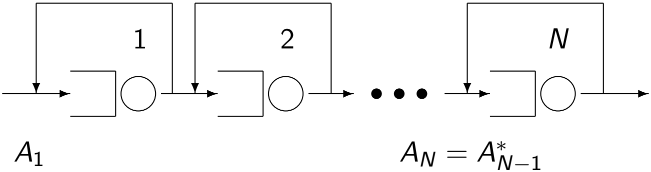

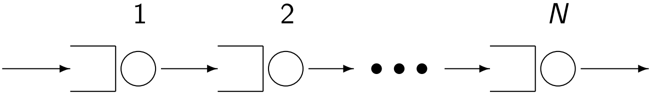

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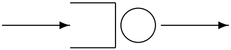

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Result with vs. without Interference

The network capacity is expressed as

$$S_1^N(t) = \left(S - \sum_{i=1}^{N} A_i^*\right) \otimes \ldots \otimes \left(S - \sum_{i=1, i \neq N-1}^{N} A_i^* - A_1\right)(t),$$

which is upper and lower bounded by

$$(S - NA_N^*)(t) \geq S_1^N(t) \geq (S - NA_1)(t).$$

The transient network capacity $\overline{C}_1^N(t)$ asymptotically converges as

$$\lim_{t\to\infty} \overline{C}_1^N(t) = \lim_{t\to\infty} \frac{S(t)}{t} - \lim_{t\to\infty} \frac{NA_1(t)}{t},$$

which results from $\lim_{t\to\infty} A_1(t)/t = \lim_{t\to\infty} A_N^*(t)/t$.

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The impact of self-interference depends on both the number of (interference) hops and the arrival rate, the larger hop number or arrival rate, the more eminent influence.

Concluding Remarks

The research is based on a series of models in the physical layer, the link layer, and the network layer.

- In the first question, a stochastic channel model is used to introduce the wireless channel capacity and a fundamental property is proposed.
- In the second question, a queueing model is used to define the performance measure and a hidden resource is discovered.
- In the third question, a queueing network is used to investigate the impact of self-interference that tends to induce a throughput bottleneck in ad hoc networks.

Further reading: Sun, Fengyou, and Yuming Jiang. “A statistical property of wireless channel capacity: theory and application.” IFIP WG 7.3 Performance 2017.