Statistical Guarantee Optimization for AoI in Single-Hop and Two-Hop FCFS Systems with Periodic Arrivals

Jaya Prakash Champati

Postdoctoral researcher, School of EECS, KTH
(Assistant Professor, IMDEA Networks Institute, Madrid)

WoNeCa 2020

Relevance of Freshness in WNCS

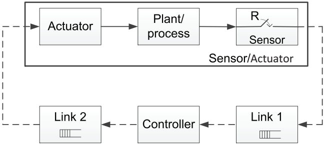

Figure: Wireless Networked Control System (WNCS)

- WNCS need to support time-critical closed-loop app.
  - Example: automation, augmented reality, power grid etc.

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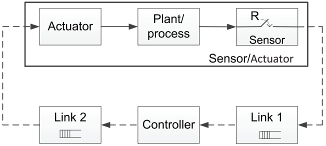

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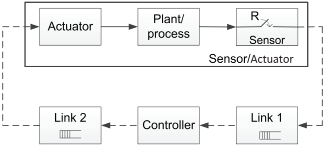

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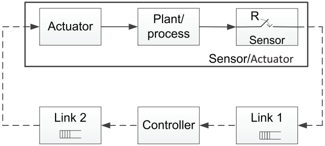

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- Minimizing end-to-end packet delay or maximizing throughput doesn't guarantee timely actuations
- **Need for a design of the system using freshness metric**

Age of Information (AoI): Freshness Metric

Definition [Kaul,2011]: Time elapsed since the most recently generated packet that is received;

- $\Delta(t) = t-$ generation time of freshest packet

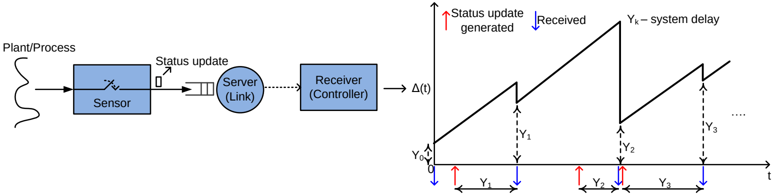


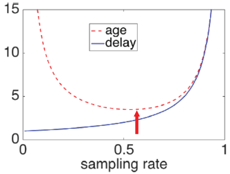
Figure: Example evolution of AoI at the receiver (Controller).

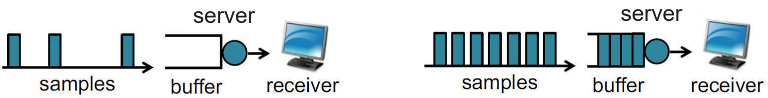
- AoI = Delay, when a packet is received
- Lower AoI implies fresh samples

AoI vs Delay

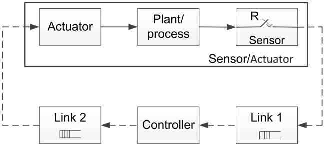
In FCFS queues, as sampling rate increases [Kaul et. al.'12]:

- AoI first decreases and then increases
- Delay increases





Problem Statement



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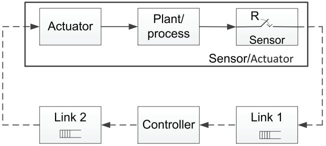

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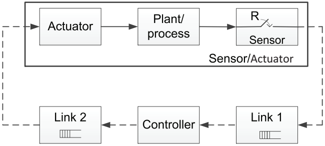

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- **QoS requirement:** End-to-end AoI at the actuator cannot exceed *age limit* d
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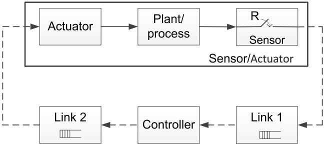

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Compute R that minimizes AoI violation probability

AoI related work

Queuing Analysis

- M/M/1, M/D/1, and D/M/1 FCFS - [Kaul12]
- LCFS - [Kaul12],[Najm16]
- No queue, unit capacity queue [Costa16], [Soysal18]
- ...

Optimization

- Average AoI - [Yates15,Huang16,Talak'18,Soysal'19,Bacinoglu'19,Talak'19]
- Optimal sampling instants, single hop - [Sun et. al.17,Champati'20]
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Our work: periodic arrivals, FCFS, tandem queuing, AoI violation probability minimization

System Model and Objective

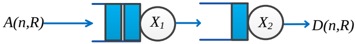

Figure: Two-hop network model.

- n - packet index, $k \in \{1, 2\}$ - node index
- Service time for packet n at node k - X_k^n (i.i.d.)
  - Heterogeneous and generally distributed
  - Mean service rate μ_k, and $\mu = min\{\mu_1, \mu_2\}$
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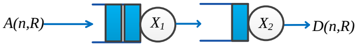

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- Packets are served using FCFS

Given $d \geq 0$, minimize the AoI *violation probability*.

$$\mathcal{P}: \quad \min_{R} \lim_{t \to \infty} \mathbb{P}(\Delta(t, R) > d).$$

Rest of the Talk

- We characterize AoI violation probability
  - In the feasibility rate region $\{\Delta(t, R) > d\} \equiv \{D(\hat{n}_R) > t\}$

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- UBMP is computationally intensive
  - α-relaxed upper bound - α-UBMP sol.
  - Chernoff upper bound - Chernoff-UBMP sol.

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- UBMP is computationally intensive
  - α-relaxed upper bound - α-UBMP sol.
  - Chernoff upper bound - Chernoff-UBMP sol.
- Numerical evaluation: different service-time distributions
  - Properties of the upper bounds
  - Compare performance of Chernoff-UBMP and α-relaxed UBMP solutions with optimal rate solutions (simulated)

Characterizing AoI Distribution

Theorem

AoI violation probability for a single-source single-receiver system under FCFS network is characterized as follows:

1. *If* $R \geq \frac{1}{d}$, $\lim_{t\to\infty} \mathbb{P}\{\Delta(t,R) > d\} = \lim_{t\to\infty} \mathbb{P}\{D(\hat{n}_R) > t\}$
2. *Else if* $R < \frac{1}{d}$, *then*

$$\limsup_{t\to\infty} \mathbb{P}\{\Delta(t,R) > d\} = 1,$$

$$\liminf_{t\to\infty} \mathbb{P}\{\Delta(t,R) > d\} = \lim_{t\to\infty} \mathbb{P}\{D(\hat{n}_R) > t\},$$

$\hat{n}_R$ *denotes the index of the first arrival on or immediately after time* $t - d$.

- $R \geq \frac{1}{d}$ is a necessary condition for existence of AoI violation probability

Equivalent Problem $\tilde{\mathcal{P}}$

- An equivalent problem to $\mathcal{P}$

$$\tilde{\mathcal{P}} : \quad \min_{\frac{1}{d} \leq R < \mu} \lim_{t \to \infty} \mathbb{P}(D(\hat{n}_R) > t)$$

  - $R < \mu$ ensures queue stability
- **Max-plus algebra**: input-output relation at link k

$$D_k(n) = \max_{0 \leq n \leq n} \{A_k(n - v, R) + \sum_{i=0}^{n} X_k^{n-i}\}$$

- Exact expression for $\mathbb{P}(D(\hat{n}_R) > t)$ is intractable
  - $D(\hat{n}_R)$ - maximum of $\hat{n}_R + 1$ correlated random variables

Upper Bound Min. Problem (UBMP)

Lemma: Given d, an upper bound for AoI violation probability is given by

$$\lim_{t\to\infty} \mathbb{P}(D(\hat{n}_R) > t) \leq \lim_{\hat{n}_R\to\infty} \sum_{v_0=0}^{\hat{n}_R} \sum_{v_1=0}^{\hat{n}_R - v_0} \Phi(v_0, v_1, R),$$

$$\Phi(v_0, v_1, R) \triangleq \mathbb{P}\left\{\sum_{i=0}^{v_0} X_2^i + \sum_{i=0}^{v_1} X_1^i > d + \frac{v_0 + v_1 - 1}{R}\right\}.$$

- $\Phi(v_0, v_1, R)$ - distribution of sum of independent rvs

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UBMP: Minimize upper bound for $R \in [\frac{1}{d}, \mu)$

- However, UBMP requires computation of infinite terms

α-UBMP

Theorem: For any $K \geq 1$, the α-relaxed upper bounded is given by $\alpha(K) \cdot \sum_{v_0=0}^{K-1} \sum_{v_1=0}^{K-1} \Phi(v_0, v_1, R)$,

$$\alpha(K) = 1 + \frac{\min_{s \in \mathcal{S}} \Psi(s, d, R, K)}{\sum_{v_0=0}^{K-1} \sum_{v_1=0}^{K-1} \Phi(v_0, v_1, R)},$$

$$\Psi(s, d, R, K) = e^{-s(d-\frac{1}{R})} M_1(s) M_2(s) \frac{(\beta_1^K(s) + \beta_2^K(s) - \beta_1^K(s)\beta_2^K(s))}{(1-\beta_1(s))(1-\beta_2(s))},$$

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α**-UBMP:** Minimize α-relaxed upper bound for $R \in [\frac{1}{d}, \mu)$.

- α-UBMP sol.: use exhaustive search on $[\frac{1}{d}, \mu)$

Chernoff-UBMP

The Chernoff upper bound is given by,

$$lim_{t\to\infty}\mathbb{P}\{D(\hat{n}_{\mathrm{R}}) > t\} \leq \min_{s\in\mathcal{S}} \Psi_2(s,d,R),$$

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- The bound is loose compared to α-relaxed UB
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- Chernoff-UBMP sol.: use bisection search on $[\frac{1}{d}, \mu)$

Numerical Evaluation

Upper Bounds: Varying R

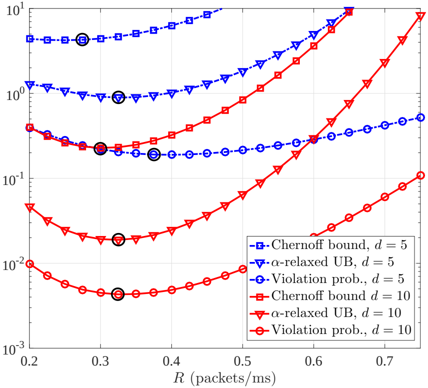
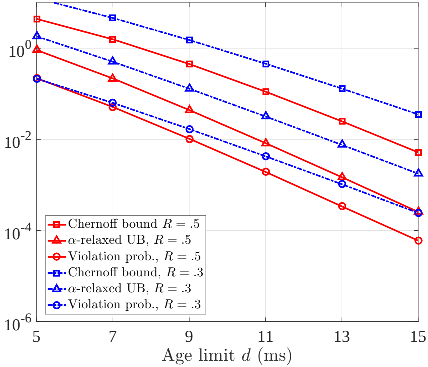


Figure: Exponential service at both links with service rate 1.

- Minimum for the upper bound and AoI violation probability occur near optimal *R*.

Upper Bounds: Varying d



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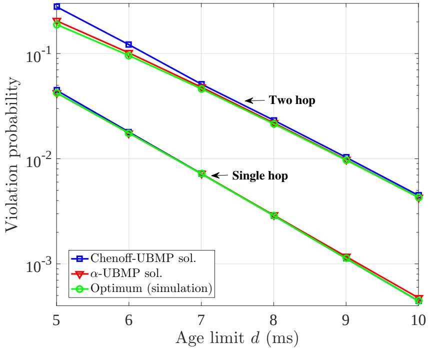
- **Key:** α-relaxed UB is almost linearly proportional to AoI violation probability

Quality of UBMP Solutions



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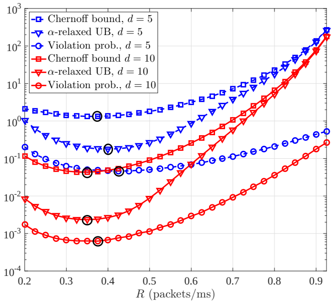
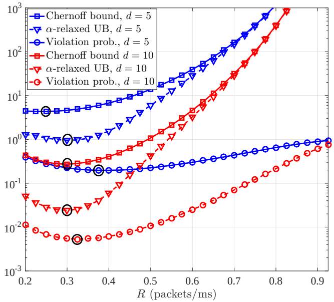


(a) Single hop



(b) Two hop

Figure: Service time PDF - $p\lambda_1 e^{-\lambda_1 x} + (1-p)\lambda_2 e^{-\lambda_2 x}$, $p = 0.91$, $\lambda_1 = 0.95$, and $\lambda_2 = 2$.

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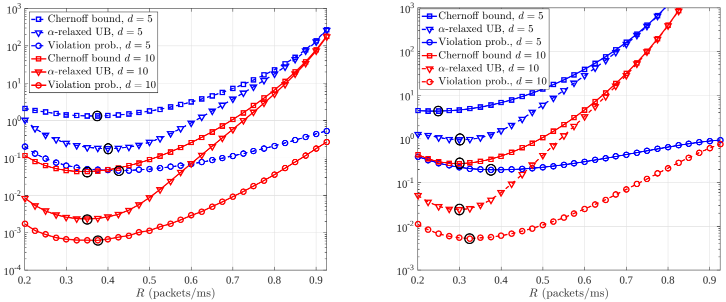

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- Similar trends for Geometric and Erlang service-time distributions.

Summary

- AoI violation probability is relevant metric for time-critical applications
- Modeled WNCS as a two-hop network and characterized the AoI violation probability
- Proposed α-UBMP and Chernoff-UBMP to solve for minimizing AoI violation probability
- Demonstrated the efficacy of the heuristic solution for different service-time distributions

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