Formalization of relations between cumulative curves and event streams: from network calculus to CPA, and back

Marc Boyer[1,*], Pierre Roux[1], Leonie Köhler [2], Borislav Nikolic [2], Rolf Ernst[2]

5th Workshop on Network Calculus (WoNeCa-5)
October 9th 2020

ONERA
THE FRENCH AEROSPACE LAB

[1]ONERA, [2]TU Braunschweig

Outline

Outline

Curve based models

NC Network calculus

- upper/lower arrival curves
- (strict) minimal (maximal) service curve
- shaping curves

RTC Real-Time calculus

- upper/lower arrival curves
- upper/lower service curves
- greedy shapers

CPA Compositional Performance Analysis

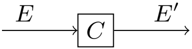
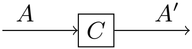
- event stream
- event distance
- busy window

- Three models
- Relation RTC $\leftrightarrow$ NC
  - equivalence [1, 2] up to technical details
- Relations CPA $\leftrightarrow$ NC
  - quite the same models of workload [3, 4]
  - different analysis methods

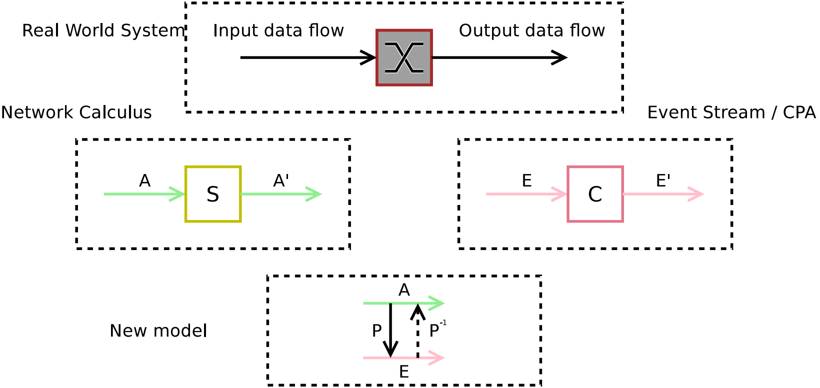
Outline

Outline

Two flow/component models

	Event Stream/CPA	Network Calculus
		
Flow model	$E(t)$: number of events up to time t	$A(t)$: amount of data up to time t
Contract $\forall t, d \geq 0$	η^+, η^-: event arrival functions $E(t+d) - E(t) \leq \eta^+(d)$ $E(t+d) - E(t) \geq \eta^-(d)$	α^u, α^l: upper and lower arrival curves $\alpha^l(d) \leq A(t+d) - A(t) \leq \alpha^u(d)$
Flow transformation	Busy window	Residual service

The global picture

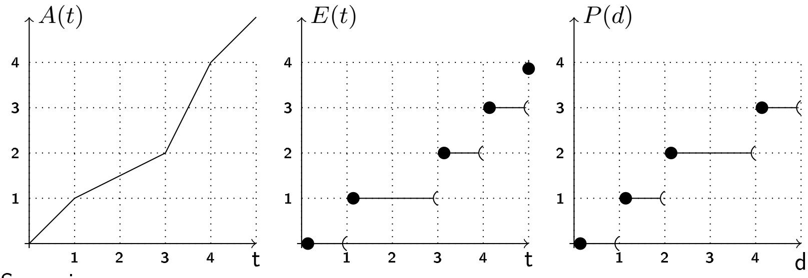


A packet function as gateway

Arrival curve	Packet count	Event count
$A : \mathbb{R}^+ \to \mathbb{R}^+$	$P : \mathbb{R}^+ \to \mathbb{N}$	$E : \mathbb{R}^+ \to \mathbb{N}$
$A(t)$: amount of data up to t	$P(d)$: number of full packets in the d first "bits"	$E(t)$: number of full packets up to t

$$\underbrace{P(\overbrace{A}^{NC})}_{[5]} = \overbrace{E}^{CPA}$$

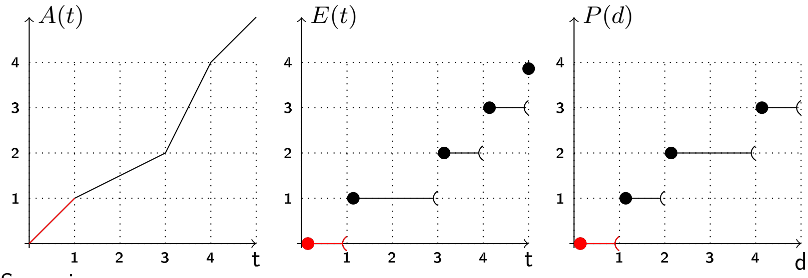
Illustration



Scenario:

- First packet: size 1, throughput 1
- Second packet: size 1, throughput 1/2
- Third packet: size 2, throughput 2
- Fourth packet: size 1, throughput 1

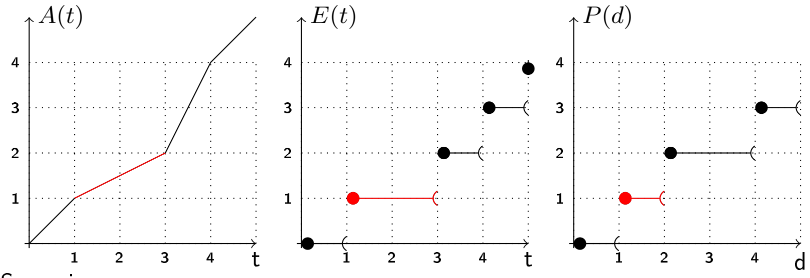
Illustration



Scenario:

- First packet: size 1, throughput 1
- Second packet: size 1, throughput 1/2
- Third packet: size 2, throughput 2
- Fourth packet: size 1, throughput 1

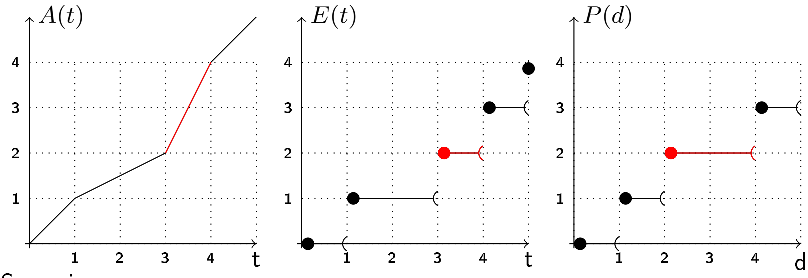
Illustration



Scenario:

- First packet: size 1, throughput 1
- Second packet: size 1, throughput 1/2
- Third packet: size 2, throughput 2
- Fourth packet: size 1, throughput 1

Illustration



Scenario:

- First packet: size 1, throughput 1
- Second packet: size 1, throughput 1/2
- Third packet: size 2, throughput 2
- Fourth packet: size 1, throughput 1

Outline

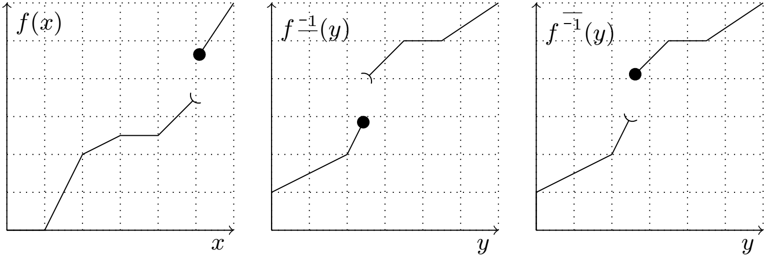
Interval Bounding Pair (IBP)

- Generalisation of arrival curves/enveloppe/event streams
- Interval Bounding Pair: renaming of arrival curves/event stream
  $\phi = (\underline{\phi}, \overline{\phi})$ is an Interval Bouding Pair (IBP) of f iff

$$\forall t, d \geq 0 : \underline{\phi}(d) \leq f(t+d) - f(t) \leq \overline{\phi}(d)$$

- Same properties than arrival curves: minimum (resp. maximum) of upper (resp. lower) arrival curves, sub/supper-additive closure, etc.

Pseudo-inverse



In [6], 25 properties on pseudo-inverses, like

$$f(x) < y \implies x \leq f^{\underline{-1}}(y), \qquad (1)$$

$$(f \circ g)^{\overline{-1}} \leq g^{\overline{-1}} \circ f^{\overline{-1}}, \qquad (2)$$

$$\overline{\phi}^{\overline{-1}}(\delta) \leq f^{\underline{-1}}(y+\delta) - f^{\underline{-1}}(y) \leq \underline{\phi}^{\underline{-1}}(\delta). \qquad (3)$$

Outline

Outline

From NC to CPA, and back

- The expected results

A	P	E

From NC to CPA, and back

- The expected results
  - from NC to CPA,

A	P	E
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\pi}, \overline{\pi})$	

From NC to CPA, and back

- The expected results
  - from NC to CPA,

A	P	E
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\pi} \circ \underline{\alpha}, \overline{\pi} \circ \overline{\alpha})$

From NC to CPA, and back

- The expected results
  - from NC to CPA,
  - and back,

A	P	E
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\pi} \circ \underline{\alpha}, \overline{\pi} \circ \overline{\alpha})$
	$(\underline{\pi}, \overline{\pi})$	$(\underline{\eta}, \overline{\eta})$

From NC to CPA, and back

- The expected results
  - from NC to CPA,
  - and back,

A	P	E
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\pi} \circ \underline{\alpha}, \overline{\pi} \circ \overline{\alpha})$
$(\underline{\overline{\pi}^{-1}} \circ \underline{\eta}, \overline{\underline{\pi}^{-1}} \circ \overline{\eta})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\eta}, \overline{\eta})$

From NC to CPA, and back

- The expected results
  - from NC to CPA,
  - and back,
  - and for completeness.

A	P	E
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\pi} \circ \underline{\alpha}, \overline{\pi} \circ \overline{\alpha})$
$(\overline{\pi}^{\underline{-1}} \circ \underline{\eta}, \underline{\pi}^{\overline{-1}} \circ \overline{\eta})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\eta}, \overline{\eta})$
$(\underline{\alpha}, \overline{\alpha})$		$(\underline{\eta}, \overline{\eta})$

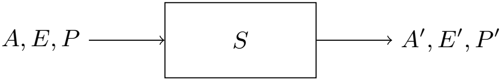
From NC to CPA, and back

- The expected results
  - from NC to CPA,
  - and back,
  - and for completeness.

A	P	E
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\pi} \circ \underline{\alpha}, \overline{\pi} \circ \overline{\alpha})$
$(\overline{\pi}^{\underline{-1}} \circ \underline{\eta}, \underline{\pi}^{\overline{-1}} \circ \overline{\eta})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\eta}, \overline{\eta})$
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\eta}_l \circ \overline{\alpha}^{\underline{-1}}, \overline{\eta}_r \circ \underline{\alpha}^{\overline{-1}})$	$(\underline{\eta}, \overline{\eta})$

Outline

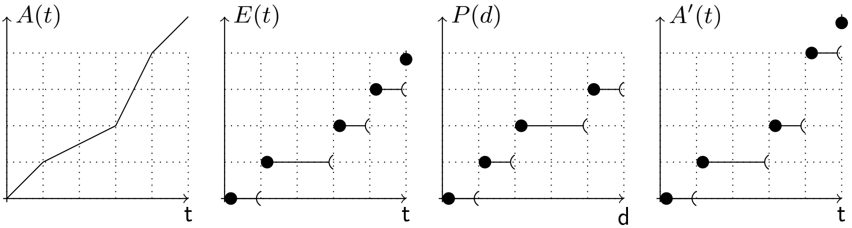
Packetizer: generalising previous results



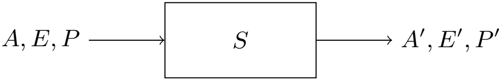
Packetizer:

- store bits, up to end-of-packet
- instantaneous packet output
- model: E, P unchanged

$$A' := P^{-1} \circ P \circ A$$
$$E' := E$$
$$P' := P$$



Packetizer: generalising previous results



Packetizer:

- store bits, up to end-of-packet
- instantaneous packet output
- model: E, P unchanged

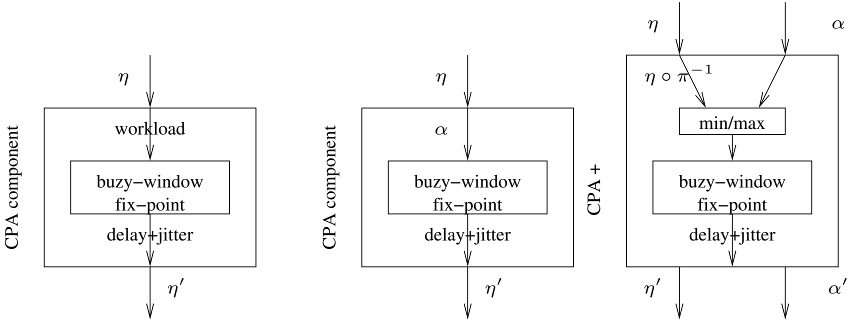
$$A' := P^{\underline{-1}} \circ P \circ A$$
$$E' := E$$
$$P' := P$$

$$\underline{\alpha}' := \overline{\pi}^{\overline{-1}} \circ \underline{\eta}$$
$$\overline{\alpha}' := \underline{\pi}^{\underline{-1}} \circ \overline{\eta}$$

Outline

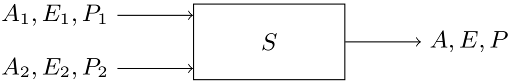
CPA integration

- Event stream: $\underline{\eta}, \overline{\eta}$
  Bounding number of events in a time interval.
- Event distance: $\underline{\delta}, \overline{\delta}$
  Bounding distance between events.
- Contributions related to curves:
  - definition of event occurence function T
  - definition of $\underline{\delta}, \overline{\delta}$ as IBP of T
  - relations between $\underline{\delta} \leftrightarrow \overline{\eta}$ and $\overline{\delta} \leftrightarrow \underline{\eta}$.
- Contributions related to analysis:
  - rewriting of busy-window analysis with "arrival curve" notations
  - adaptation to variable packets/workload


CPA component
η
workload
buzy−window
fix−point
delay+jitter
η'
CPA component
η
α
buzy−window
fix−point
delay+jitter
η'
CPA +
η
α
$\eta \circ \pi^{-1}$
min/max
buzy−window
fix−point
delay+jitter
η'
α'

Outline

Aggregation



Aggregation:

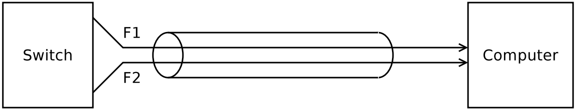
- mix of flows
- "sum" of flows is a flow
- no delay

$$A := A_1 + A_2$$
$$E := E_1 + E_2$$
$$P(A_1 + A_2) := P(A_1) + P(A_2)$$

$$\underline{\alpha} := \underline{\alpha}_1 + \underline{\alpha}_2$$
$$\underline{\eta} := \underline{\eta}_1 + \underline{\eta}_2$$
$$\underline{\pi} := \lfloor \underline{\pi_1} \underline{*} \underline{\pi_2} \rfloor$$

$$\overline{\alpha} := \overline{\alpha}_1 + \overline{\alpha}_2$$
$$\overline{\eta} := \overline{\eta}_1 + \overline{\eta}_2$$
$$\overline{\pi} := \lceil \overline{\pi_1} \overline{*} \overline{\pi_2} \rceil$$

Case study



- Two data flows, F_1, F_2, from S to C
- Using a link of throughput 1

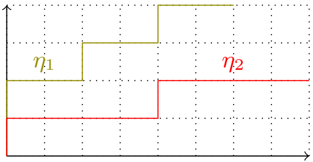
Flow	Packet size	Burst	Throughput	$\overline{\alpha}_i$	$\overline{\pi}_i$
F_1	1/2	1	1/4	x/4 +1	$\lceil 2x \rceil$
F_2	1	1	1/4	x/4 +1	$\lceil x \rceil$

- Goal: evaluation of the packet throughput
  - $F = F_1 + F_2$
  - what is $\overline{\eta}$?
  - challenge: modelling the link shaping

Packet throughput: no shaping

No shaping :

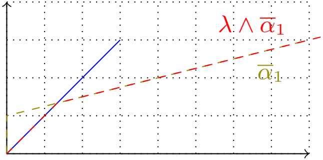
- $\overline{\eta}_1 = \overline{\pi}_1 \circ \overline{\alpha}_1 = \left\lceil \frac{x}{2} \right\rceil + 2$
- $\overline{\eta}_2 = \overline{\pi}_2 \circ \overline{\alpha}_2 = \left\lceil \frac{x}{4} \right\rceil + 1$
- $\overline{\eta} \leq \overline{\eta}_1 + \overline{\eta}_2$

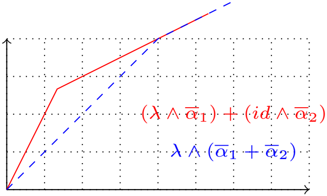


Packet throughput: with shaping

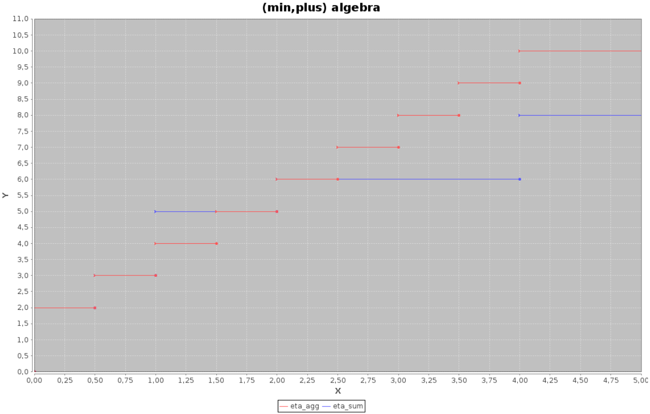
Link throughput: $\lambda(t) = t$

- Shaping reduces data throughput
  - for each flow,
    $\overline{\alpha}_i^s = \lambda \wedge \overline{\alpha}_i$
  - for the aggregate flow:
    $\overline{\alpha}_{1+2}^s = \lambda \wedge (\overline{\alpha}_1 + \overline{\alpha}_2)$
- Impact on packet throughput
  - per flow: $\overline{\eta}_i^s = \overline{\pi}_i \circ \overline{\alpha}_i^s$
  - aggregate flow:
    $\overline{\eta}_{1+2}^s = \lceil \overline{\pi}_1 \ast \overline{\pi}_2 \rceil \circ \overline{\alpha}_{1+2}^s$
  - both $\overline{\eta}_1^s + \overline{\eta}_2^s$ *and* $\overline{\eta}_{1+2}^s$ are packet throughput bounds





Numerical results



- the shaping only affects start of curve
- the simple method has better long term throughput

Outline

Conclusion

- A step forward in modelling packets
- Some theoretical results
- Aggregation result still disappointing on real examples
- Large implementation effort

Outline

References

[1] A. Bouillard, L. Jouhet and E. Thierry,
Service curves in Network Calculus: dos and don'ts,
Research Report INRIA n^o RR-7094 (2009).

[2] A. Bouillard, M. Boyer and E. Le Corronc,
Deterministic Network Calculus – From theory to practical implementation,
No. ISBN: 978-1-119-56341-9. Wiley (2018).

[3] M. Boyer and P. Roux,
Embedding network calculus and event stream theory in a common model,
In *Proc. of the 21st IEEE Int. Conf. on Emerging Technologies and Factory Automation (ETFA 2016)*, Berlin, Germany (September 2016).

[4] L. Köhle, B. Nikolić and M. Boyer,
Increasing Accuracy of Timing Models: From CPA to CPA+,
In *Proc. of the Design, Automation and Test in Europe Conference and Exhibition (DATE)*, Florence, Italy (March 2019).

[5] A. Bouillard, N. Farhi and B. Gaujal,
Packetization and Packet Curves in Network Calculus,
In *Proc. of the 6th International Conference on Performance Evaluation Methodologies and Tools (ValueTools 2012)*, Cargese, France (October, 9–12 2012),
Invited Paper.

[6] M. Boyer and P. Roux,
A common framework embedding network calculus and event stream theory,
working paper / hal-01311502.

[7] A. Bouillard, N. Farhi and B. Gaujal,
Packetization and Aggregate Scheduling,
Technical report INRIA n^o 7685 (2011).