An Analytical Model of a Hybrid Network of Static and Mobile Sensors

Abrahale Tewelde, Wint Yi Poe, Jens B. Schmitt
University of Kaiserslautern, Germany
atewelde@rhrk.uni-kl.de, {poe,jschmitt}@cs.uni-kl.de

ABSTRACT

There is a trend of conventional static WSNs towards hybrid static and mobile WSNs which can be seen in many recent protocols and designs. By enabling node mobility in conventional WSNs, the number of static sensors can be reduced effectively. Yet, what is still missing about a hybrid WSN is the optimal proportion of mobile and static sensors that can achieve the same network performance as a conventional WSN. In this paper, we propose an analytical model for hybrid WSNs, in particular, dealing with the area coverage issue while minimizing energy consumption. We first present the optimal proportion of mobile and static sensors in order to guarantee a full coverage with high probability under a uniform random node distribution. A mobility strategy is then developed which minimizes energy consumption due to node mobility. Finally, we evaluate the performance of the proposed model and mobility strategy using a discrete event network simulation framework based on OMNeT++. The simulation results validate the analytical solution.

Categories and Subject Descriptors

C.2 [**Computer-Communication Networks**]: Network Architecture and Design—*Wireless communication*; C.4 [**Performance of Systems**]: Modeling techniques

General Terms

Algorithm, Design, Performance

Keywords

Hybrid WSNs, Optimal Proportion of Static and Mobile Sensors, Mobility Algorithm, Coverage, Energy Consumption.

AINTEC'13, November 13–15, 2013, Chiang Mai, Thailand.

1. INTRODUCTION

The research focus of hybrid WSNs composed of mobile and static sensors is increasing due to the high general interest in mobility as a design parameter in WSNs [3, 9, 12, 13]. Generally, mobile sensors reduce the number of static sensors and can improve network performance. Besides, a limited mobility approach is considered in hybrid WSNs, because an excessive number of mobile sensors can increase cost and complexity of other network functions as, for example, routing.

The quality of service of a WSN can be measured by various performance metrics depending on its applications. A crucial and fundamental metric of WSN applications is area coverage. Typically, the coverage determines the quality of data which plays a vital role in most WSN applications. The point is that the accuracy of data is directly proportional to the monitoring area of the network covered by sensors. A sensor can monitor events occurring within a certain distance from itself which can be defined as its sensing range. Due to environmental obstacles or electromagnetic waves a sensor may be unreliable with respect to monitoring its sensing range. Yet, the required level of redundancy, i.e., the notion of k-coverage, where $k \geq 1$ is the minimum number of sensor(s) by which each point in the sensor field is covered, varies in applications. Often, a full coverage (i.e., 1-coverage) is a sufficient as well as necessary condition in many WSN applications. A critical aspect that determines coverage is sensor deployment.

Among various deployment strategies a random node distribution is preferable owing to its simplicity, inexpensiveness, and feasibility in being deployed in harsh environments and large-scale networks. Obviously, the performance of a random deployment relies very much on the node density. In [8], the authors addressed a probabilistic k-coverage model that can determine the appropriate number of sensors to provide k-coverage of a region under mostly sleeping sensor nodes. The authors also claimed that a uniform random deployment outperforms both the grid and the Poisson distribution placements for k-coverage. Similarly, [14] proposed a sufficient and necessary conditions for k-coverage. While [8, 14] focus on conventional WSNs, a related work of probabilistic field coverage in a hybrid WSN has been studied in [11] where static sensors are activated according to some activation schedule and a fixed number of mobile sensors are always in motion to assist the static sensors. In a hybrid network, it is natural to ask, given a certain number of sensors, what would be the best proportion of the number of mobile sensors to the number of static sensors in the network to acquire a full coverage with high probability. The authors in [12] proposed a hybrid network structure which uses a static sensor density of $\lambda = 2\pi k$ and a mobile sensor density of $\frac{\lambda}{\sqrt{2\pi k}}$ to provide k-coverage for small-scale networks under the constraint of a maximum moving distance for mobile sensors. Similarly, we tackle the problem of finding the best proportion of static and mobile sensors, yet without constraints on the moving distance.

In this paper, we introduce an analytical model of a hybrid sensor network to provide the optimal proportion of static and mobile sensors that guarantees a full coverage with high probability. We first study the lower and upper bound node densities that guarantee a full coverage. Based on this, the relationship among static, mobile, and total node densities can be established. In a hybrid network of random deployment, mobile sensors relocate their positions in order to enhance coverage performance after the initial deployment. In [5], the authors claimed that a greedy algorithm is a good one because it can pick the sets that cover the maximum uncovered elements with an approximation ratio of $1 - \frac{1}{e}$ compared to the maximum approximation ratio of $1 - \frac{1}{e} + e$. A similar approach has been considered in our mobility strategy. From repositioning of mobile sensors to target locations arises another optimization problem. In fact, it is an assignment problem which deals with the question how to assign n resources (e.g., mobile sensors) to n tasks (e.g., targets). As a decision criterion, we used the energy consumption owing to node mobility in the assignment of mobile sensors to targets. Among various assignment problems, we took linear sum assignment problem (LSAPs) [2] and bottleneck assignment problems (LBAPs) [4] into consideration in our mobility algorithm.

2. PRELIMINARIES

2.1 Network Model and Assumptions

We consider a hybrid WSN consisting of N sensors which are deployed in a square region according to the following models and assumptions:

- N sensors are identically and independently uniform distributed in a square deployment region with area A having a side length of $\sqrt{A}$.
- Assume that V is the set of sensors with $|V| = |V_S| + |V_m| = N$ where $V_s = \{v_{s_1}, v_{s_2}, ..., v_{s_k}\}$ and $V_m = \{v_{m_1}, v_{m_2}, ..., v_{m_{N-k}}\}$ are the set of static and mobile sensors, respectively.
- Let d_{ip} be the Euclidean distance between node $i \in V$ and a point p in $[0, \sqrt{A}] \times [0, \sqrt{A}]$. For every point p, a full coverage of the network is guaranteed iff: $\exists\, d_{ip} \leq r_s$, where $i \in V$ and r_s is a disc-based sensing range.
- The node density λ is the sum of static node density λ_s and mobile node density λ_m, which is $\lambda = \lambda_s + \lambda_m$.

In addition, the locations of sensors are known a priori. Initially, all nodes including mobile sensors are supposed to be scattered randomly. For example, such a placement can result from throwing sensor nodes from an airplane. Mobile sensors are equipped with locomotion capabilities and are able to move anywhere in the deployment region after the initial deployment. We focus on the energy consumption due to node mobility. We define the node density with respect to coverage performance. In particular, the node density is defined as the area of the coverage disc to the area of a unit cell. Details can be seen in Section 3.1.

2.2 Problem Definition

The main problems we address in this paper are:

1. Given a hybrid WSN consisting of N sensors, which proportion between static and mobile sensors would be optimal to guarantee a full coverage with high probability?
2. For mobile sensors, which mobility strategy would maximize area coverage of the network while minimizing energy consumption?

2.3 Contributions

We model and analyze a hybrid WSN under a random deployment strategy. Our major contributions are:

- We characterize the coverage node density under a random deployment strategy which is sufficient to guarantee a full coverage with high probability. ($\rightarrow$ Section 3)
- We introduce an analytical model for the best proportion of static and mobile sensors. ($\rightarrow$ Section 3)
- We then derive a mobility algorithm for mobile nodes to accomplish the desired coverage goal while minimizing energy consumption owing to node mobility. ($\rightarrow$ Section 4)
- We finally conducted thorough experiments using discrete-event simulation to validate the analytical solutions and to evaluate the proposed mobility algorithm. ($\rightarrow$ Section 5)

3. HYBRID WSNS

The mobile sensors in a hybrid WSN have the ability to reduce the node density and to enhance the network performance. Mobile sensors can move to a required position such that they can achieve a better performance compared to static sensors which do not have the ability to move. An obvious example is to maximize network coverage by mobile sensors that may be left uncovered by purely static sensors. Simultaneously, repositioning of mobile sensors can generate a balanced node distribution by moving away from an uneven node distribution.

On one hand, deploying only mobile sensors can get the network performance similar to a deterministic deployment by repositioning of mobile sensors to a desired deployment. For example, a triangular tiling of N sensors, which is known to be an optimal deployment for coverage performance, can be obtained by relocation of N mobile sensors which are deployed randomly to a similar triangular placement. Without consideration of geographical impacts, a sufficient condition to achieve the optimal deployment of having N sensors is N mobile nodes in the non-deterministic deployment.

On the other hand, an excessive number of mobile sensors can increase the complexity of other network functions due to dynamic topology changes. These include routing complexity, data retransmission, and communication overheads, etc. Moreover, locomotion capability is very expensive for a large number of mobile sensors. Hence, a hybrid network with limited node mobility can find a good compromise between cost and performance issues. Now the question arises: what is the optimal proportion of static and mobile sensors?

To deal with this question, initially, we focus on area coverage. A sufficient condition of coverage performance in most WSN applications is that a network can guarantee to detect an event everywhere in the network, i.e., a full coverage. In [10], the authors addressed that a random node deployment has a diverse relative frequency of k-coverage. Due to the randomness, a necessity of coverage performance is guaranteeing a full coverage with high probability.

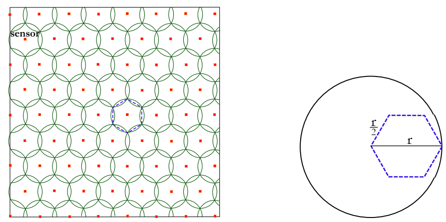
In this section, we introduce an analytical model for the best ratio of static and mobile sensors for a given number of sensors N to attain the aforementioned coverage performance. To formulate the best proportion of static and mobile sensors, the lower and upper bound node densities are required. We develop an approach for determining the node density of a random deployment in the following subsection.

3.1 Node Density of Random Deployment

Typically, node density can be defined as the number of nodes per unit area. Without losing this standard notion, we characterize node density in relation to coverage as the area of the coverage disc to the area of a unit cell. Therefore,

$$\lambda = \frac{\text{area of a coverage disc}}{\text{area of a unit cell}}. \quad (1)$$

In this definition, the area of a unit cell is the smallest corresponding coverage area of a sensor node. In the general case, it is equivalent to $\frac{A}{N}$ where A is a network area and N is the number of sensors. In a deterministic network as, for example a tiling-based deployment strategy [10], a unit cell can be constructed by connecting the centroids of the surrounding polygons of a node without consideration of boundary conditions. For instance, a hexagon cell for triangular tiling is illustrated in Figure 1(a).



(a) (b)

Figure 1. (a) Triangular tiling deployment and (b) the smallest coverage disc.

As mentioned above, we focus on a full coverage node density under a random deployment strategy. In a random deployment, sensors are inprecisely placed in a region without knowing their locations in advance. As a result, a random deployment cannot guarantee coverage performance. It is clear that the number of sensors required to accomplish a full coverage is much more than that of the optimal deployment. In [14], the node density $\lambda = log(A)+(k+2)loglogA+c(A)$ with $c(A) \to +\infty$ is sufficient to guarantee k-coverage over the region A, which, in fact, is an unbounded density compared to an optimal deterministic deployment.

Being the simplest method and feasible in harsh environments as well as large-scale networks, a better bound on the node density for a random deployment should be studied. To that end, we introduce an approach of the coverage node density which is analogous to the node density of the optimal deployment. Primarily, we study the optimal deployment pattern that fulfills a full coverage. In literature, it has been shown that placing discs at the vertices of triangular lattice, as shown in Figure 1(a), is optimal with respect to the number of discs needed to achieve a full coverage [6]. The optimal deployment can be constructed as a tiling of equilateral triangles with side length $\frac{3}{\sqrt{3}}r_t$ and height of $\frac{3}{2}r_t$, where r_t is the radius of coverage disc and each vertex hosts a sensor. As we discussed above, the unit cell of a triangular tiling is a regular hexagon of area $\frac{3\sqrt{3}}{2}r_t^2$.

Based on this unit cell of optimal deployment, we model the lower bound node density for random deployment to achieve a full coverage. The idea is finding the smallest coverage disc in which a unit cell can be fully covered by that disc wherever a sensor is placed in the cell. An extreme case is illustrated in Figure 1(b). We can see that a unit cell with side length $\frac{r}{2}$ can be fully covered by a sensor having a minimum coverage disc r. Recall that the coverage node density, λ, is the area of the sensing disk divided by the area of the smallest corresponding cell. Therefore,

$$\lambda = \frac{\pi r^2}{\frac{3\sqrt{3}}{8} r^2} = \frac{8\pi}{3\sqrt{3}} \approxeq 4.837. \quad (2)$$

Note that, in the optimal deployment, a cell can be fully covered only if a sensor is placed at the center of a cell. Obviously, the optimal deployment has a lower bound node density of

$$\lambda_o = \frac{\pi r_t^2}{\frac{3\sqrt{3}}{2} r_t^2} = \frac{2\pi}{3\sqrt{3}} \approxeq 1.21.$$

The node density of the optimal deployment is $\Theta(1)$ which is independent of the network area without consideration of boundary effects. If the boundary condition is taken into consideration, the sufficient node density becomes $\lambda_o \geq \frac{2\pi}{3\sqrt{3}}$. In contrast, the node density in Equation (2) is formulated based on a single unit cell, i.e., a regular hexagon cell. We apply the same concept for the network of area $A \geq \frac{8\pi}{3\sqrt{3}}$ by dividing it into regular hexagon cells. With a Poisson distribution, we can check a full coverage performance with parameter $\mu = \lambda$. The probability that a cell is fully covered is 99.2%. The alert reader will notice that the proposed node density can guarantee a full coverage for a reasonable network area A. As the area of the network increases, the probability of a full coverage decreases very slowly. This slow decrease of full coverage probability tends to zero if the area approaches to infinity. This is proved in the following lemma.

Lemma 1: If $A \to \infty$, then the density λ cannot guarantee a full coverage for a region of area A.

PROOF. Assume that a network of area A has a number of sensors n with node density λ according to (2). A network is divided into $m = \frac{A}{a}$ cells where a is the area of a unit cell. Let P_i^1 and P_i^0 be the probability that the sensor is placed at cell i and the probability that the sensor is not placed at cell i. In a uniform random deployment, a node has equal likelihood to be positioned in any cell $i = 1, ..., m$. Hence, the probability of a sensor to be positioned at cell i is $\frac{1}{m} = \frac{a}{A}$. The correlation between the node density λ, the number of sensors n, and the area of the network A is $n = \lambda A$. Let $C_i(A)$ be the probability that a cell i in a region A is covered by one out of n sensor that is placed at cell i. Thus,

$$C_i(A) = 1 - (1 - P_i^1)^n = 1 - (1 - \frac{a}{A})^{\lambda A}$$

If $A \to \infty$, $C_i(A) \to 0$ since $\lambda > 0$. □

From the proof of Lemma 1, the coverage probability is a decreasing function when increasing the network size A. This shows that the node density has to increase if the monitored area increases in order to maintain the same coverage performance. Thus, we consider a slowly increasing function in our node density model.

A similar approach has been introduced in [8, 14]. Kumar et. al [8] studied the k-coverage problem for a network of mostly sleeping sensors n. The authors showed that if there exists a slowly increasing function $f(A)$ but slower than $loglog(A)$ and $f(A) \to \infty \,\text{if}\, A \to \infty$, the node density $np\pi r^2 \geq log(np) + f(A)(1 + \sqrt{plog(np)}) + kloglog(np)$ provides k-coverage with high probability where r is the sensing range and p is the active node probability. In [14], the authors proposed a sufficient and necessary condition for k-coverage. Assume a function $c(A)$ is a monotonically slowly increasing function. Then the node density, $\lambda = log(A) + (k+2)loglog(A) + c(A)$, guarantees k-coverage if $c(A) \to \infty \,\text{when}\, A \to \infty$ and $c(A)$ grows slower than $loglog(A)$.

Based on [8, 14], we enhance our model for any network size A by taking into consideration a slowly increasing function $f(A)$ which tends to $\infty \,\text{as}\, A \to \infty$. Finally, Equation 2 becomes

$$\lambda_u = \frac{8\pi}{3\sqrt{3}} + f(A). \quad (3)$$

A candidate for a slowly increasing function $f(A)$ is $\sqrt{loglog(A)}$.

Lemma 2: The node density λ_u guarantees a full coverage for any network size A with a high probability.

PROOF. We are required to prove that if $A \to \infty \, then \, C_i(A) \to 1$. From Lemma 1, a full coverage probability of cell i is $C_i(A) = 1 - (1 - \frac{a}{A})^{\lambda A}$. By substituting λ_u,

$$\lim_{A\to\infty} C_i(A) = \lim_{A\to\infty} \Big(1 - (1 - \frac{a}{A})^{\lambda_u A}\Big) = 1 - \lim_{A\to\infty} (1 - \frac{a}{A})^{\lambda_u A}$$

Let $a = 1$ and $y = -A$, then

$$\begin{aligned}\lim_{A\to\infty} C_i(A) &= 1 - \lim_{A\to\infty} (1 + \frac{1}{y})^{-\lambda_u y} \\ &= 1 - \lim_{A\to\infty} \Big((1 + \tfrac{1}{y})^y\Big)^{-\lambda_u} \\ &= 1 - \lim_{A\to\infty} e^{-\lambda_u}\end{aligned}$$

And we know that $\lambda_u \to \infty \, when \, A \to \infty$, thus,

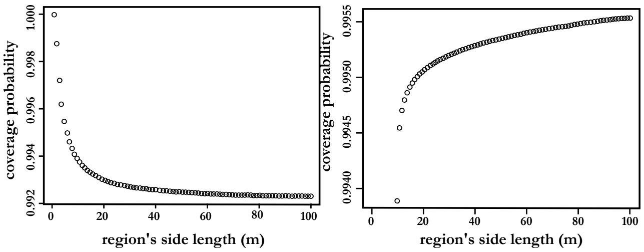
$$\lim_{A\to\infty} C_i(A) = 1 - e^{-\infty} \approxeq 1.$$

□

A comparison of node densities between Equation 2 and 3 is presented in Figure 2.

3.2 Optimal Proportion of Static and Mobile Sensors

In this section, we discuss the lower and upper bound node density in order to formulate the best proportion of static and mobile node densities. As we stated before, the node density of a hybrid network is the sum of static and mobile node densities, $\lambda = \lambda_s + \lambda_m$.



(a) $\lambda = 4.84$ (b) $\lambda_u = 4.84 + \sqrt{loglog(A)}$

Figure 2. Deployment region vs. coverage probability of (a) λ and (b) λ_u.

3.2.1 Lower bound node density

The lower bound node density for hybrid WSNs is the case where all sensors are mobile. This is due to the fact that mobile sensors can deploy themselves in an equivalent way to an optimal deployment given that there is a mobility algorithm to do so. Hence, the lower bound node density is identical to the node density of the optimal deployment $\lambda_o = \frac{2\pi}{3\sqrt{3}}$ and the total node density becomes $\lambda = \lambda_m = \lambda_o$, where $\lambda_s = 0$.

3.2.2 Upper bound node density

The upper bound node density for a hybrid WSNs is the case where all sensors are static. The challenges related to the upper bound node density are discussed in Section 3.1 where we provide a sufficient node density λ_u for random deployment. In all static sensors network, $\lambda_m = 0$ and thus $\lambda = \lambda_s = \lambda_u$.

3.2.3 Correlation between static and mobile node density

By comparing the lower and upper bound density, we can see the impact of mobile sensors with respect to coverage performance. It is due to the fact that the existence of one mobile sensor in the network can heal uncovered "holes in the network" which can be acquired by more than one static sensors that are deployed randomly. We derive the proportion of static and mobile node density in a way that λ_u and λ_o have the ability of guaranteeing a full coverage with high probability. Then the ratio of static to mobile sensors becomes $\lambda_u : \lambda_o = \frac{8\pi}{3\sqrt{3}} + f(A) : \frac{2\pi}{3\sqrt{3}} = 4 + \frac{3\sqrt{3}}{2\pi} f(A)$. Hence, we can determine that one mobile sensor is approximately equivalent to at least 4 static sensors, $\lambda_s \approxeq 4\lambda_m$.

From this relation, if one parameter out of λ, λ_s, and λ_m is given, the other two parameters can be computed optimally. In the same way, we can compute two parameters of n, n_s, and n_m if one out of these parameters and the deployment region A is known a priori. A summary of all these relationships is presented in Table 1.

4. MOBILITY STRATEGY

In Section 3, we proposed the optimal ratio of static and mobile sensors. The next issue that has to be solved in a hybrid WSN is how to relocate the mobile portion of sensors into the optimal places. We assume that the mobility algorithm is administered by a base station or a control center. In reality, the optimal number of mobile sensors produced by the analytical model may be lower than the required number of mobile sensors to cover the network area completely. Therefore, the main objective of the mobility strategy is to maximize uncovered holes so that the resulting node placement guarantees a full coverage with high probability.

Table 1. Relations among parameters.

Correlations
The total node density to guarantee a full coverage, $\lambda = \lambda_s + \lambda_m$
The optimal proportion of static node density, $\lambda_s = \frac{\lambda_u}{\lambda_o}(\lambda_o - \lambda_m)$
The optimal proportion of mobile node density, $\lambda_m = \frac{\lambda_o}{\lambda_u}(\lambda_u - \lambda_s)$
Total number of sensors, $n = n_s + n_m$
The number of static sensors, $n_s = \frac{n_u}{n_o}(n_o - n_m)$
The number of mobile sensors, $n_m = \frac{n_o}{n_u}(n_u - n_s)$
The relationship among λ, A, n, and r is $\lambda = \frac{n\pi r^2}{A}$.

4.1 Maximum Coverage Problem

Given a region of area A with a number of static sensors n_s, there exists a set of uncovered cells $U = \{c_1, c_2, ..., c_k\}$ in A. A number of mobile sensors $m = |S|$ is given to fully cover the points in U. The objective function is

$$max. |C|$$

where $C \subseteq U$ such that

$$\forall s_j \in M \quad max. \left| \bigcup_{c_i \in U} c_i \right|.$$

The above constraints state that each mobile sensor $s_j \in M$ has to maximize the uncovered cells $c_i \in U$ so that the total number of uncovered cells will be maximized. In [3], the authors reduced a deployment problem into a set covering problem. They used a well-known greedy algorithm [5] where the objective is to maximize coverage. A similar approach has been addressed in our mobility strategy but now we also take into consideration the cost issue due to node mobility from their initial places to the optimal positions.

As we discussed in Subsection 3.1, the network is subdivided into hexagonal cells. The algorithm begins to select subsets of uncovered cells where each subset has the biggest set of uncovered cells which are within a circle of radius r. Each uncovered cell is assigned a weight. The weight is defined as the number of uncovered cells covered if a mobile sensor is placed on that cell. The set of uncovered cells or targets (T) can have a single or multiple elements having the same highest weight.

The next challenge is to find a mobile sensor for the selected target(s). If a mobile sensor moves from its original position to a target location, new uncovered hole(s) may be generated. Thus, a mobile sensor has to move only if its leaving can minimize the uncovered holes. We assign the weight to each mobile sensor as we

Algorithm 1 MaxCoverage-MobileSensor-Reposition.

```
Given: A set of mobile sensors S = {s_1, s_2, ..., s_m} and a set of uncovered cells U = {c_1, c_2, ..., c_k} in the deployment region A
while (S ≠ φ || U ≠ φ) do
1. Find a set of uncovered cells or target(s) T ⊆ U with the highest weight
 for i = 1, ..., k
  (i). Assign a weight to each uncovered cell
  (ii). Sort the uncovered cells by their weights
  (iii). Choose the uncovered cell (targets T) with the highest weight
 end for
2. Find a candidate mobile sensor s_j ⊆ S with the lowest weight
 for j = 1, ..., m
  (i). Assign a weight to each mobile sensor
  (ii). Sort the uncovered cells by their weights
  (iii). Choose the mobile sensor(s) s_j ∈ M with the lowest weight
 end for
3. if (|T| > 1 || |M| > 1)
   Choose M1 or M2
   M1: Minimizing total distance traveled
   M2: Minimizing maximum distance traveled
   Return an assignment of s_j to T_i
 else
   Move s_j to T_i
 end if
4. Set U = U \ T, U = U ⋃ U', and S = S \ M
5. Repeat
end while
Return a set of cells C = ⋃_{c_i ∈ T} c_i where C ⊆ U as targets for mobile sensors s_j ∈ S
```

did in the uncovered cells. In this case, the mobile sensor(s) with the lowest weight is selected for the target(s) T. The number of mobile sensors with the lowest weight can be multiple candidates. For multiple targets and multiple candidate mobile sensors, we propose two selection methods based on different energy consumption goals: minimizing total distance traveled and minimizing maximum distance traveled which will be discussed in Subsection 4.2. By applying one of these two methods, candidate mobile(s) is assigned to the selected target(s) and both of them are removed from the original sets. Any new uncovered cells produced by repositioning of mobile sensors U' will be added into the set U. This procedure continues until either S or U is empty. The algorithm is presented in Algorithm 1.

4.2 Minimizing Energy Consumption

After computing step 1 and 2 in Algorithm 1, the next challenge is to assign the mobile sensor(s) to target(s) locations. In this case, the objective of mobile sensors is an efficient way of movement from the current locations to the target positions. Here an efficient movement means a movement with minimum energy consumption and minimum distance traveled. If both M and T have a single element, the mobile sensor will move to the target location. For the case where a single element is in either M or T, it will be assigned to the best element of the remaining set based on M1 or M2 as shown in Algorithm 1. For multiple return values in both M and T, an efficient method for the optimal mapping of mobile sensors to targets is required. We highlight the case of multiple elements in both M and T and assume that $\|T\| = \|M\|$. The assumption can be relaxed by selecting randomly from a larger set if $\|T\| \neq \|M\|$. Then the problem is for a given set of mobile sensors $M = \{s_1, s_2, ..., s_p\}$ and a set of targets $T = \{c_1, c_2, ..., c_p\}$, what is the optimal assignment of sensors to targets with respect to minimizing energy consumption. We reduce the minimizing energy consumption problem into a minimizing distance problem under the assumption that the energy consumption for locomotion is linearly proportional to the moving distance. In the following, we introduce two approaches of minimizing energy consumption.

Algorithm 2 Sensors-Targets-Assignment (M2).

```
Given: a cost matrix A of p × p
Begin:
min:=min{a_{i,j}}, max:=max{a_{i,j}}
while min<max do
  mid:=mid{a_{i,j}}
  min_max=max
  while mid> 0 do
    if (Is_Feasible (A,mid))
      max=mid, min_max=mid
    else
      min=mid
      mid=mid{a_{i,j}}
    end if
  end while
end while
Return min_max
Select a matching by choosing a set of zeros
Apply the matching to the original matrix A
.............................................................................................
Is_Feasible(A,mid)
while ∃ a_{i,j} ≤ mid do
a_{i,j} := 0
  if ∀i, i', j, j' ∃a_{i,j}, a_{i',j'} = 0 such that i! = i' and j! = j'
    return false
  else
    return true
  end if
end while
```

Minimizing total distance traversed (M1).
The goal is to minimize the sum of the shortest distances d_{ij} from each mobile sensor $s_i \in M$ to the target $c_j \in T$. The problem is similar to the well known Linear Sum Assignment Problem (LSAP) [2] where n sources need to be assigned to n tasks on a 1-to-1 basis while minimizing the total cost. In our problem, the cost function is the Euclidean distances between mobile sensors and targets. The optimization problem can be formulated as

$$min. \sum_{i,j \in M,T} \sum a_{i,j}\, x_{i,j}$$

subject to

$$\sum_{i,j \in M,T} \sum x_{i,j} = 1$$

$$a_{i,j} \geq 0.$$

Minimizing maximum distance traversed (M2).
The goal is to minimize the maximum distance d_{ij} from any mobile sensor $s_i \in M$ to its target $c_j \in T$. This problem is comparable to a linear bottleneck assignment problem (LBAP) [4] where n sources need to be assigned to n tasks on a 1-to-1 basis while maximal cost among individual assignment is minimized. The cost function is the Euclidean distances between mobile sensors and

targets. The corresponding optimization problem can be defined as:

$$min. \max_{i,j \in M,T} a_{i,j} \, x_{i,j}$$

subject to

$$\sum_{i,j \in M,T} \sum x_{i,j} = 1$$

$$a_{i,j} \geq 0.$$

The value of $x_{i,j}$ is 1 if mobile sensor s_i is assigned to target c_j and $a_{i,j}$ is the cost matrix which is the Euclidean distance between the current mobile sensor s_i to target c_j. We apply the Hungarian method [7] which is used to find the optimal assignment for a given cost matrix. Due to space limitations, we highlight the minimizing maximum distance algorithm which is based on Hungarian method with a threshold algorithm and a binary search algorithm [2], as described in Algorithm 2. In brief, the algorithm sorts the cost matrix in ascending order and sets the upper and the lower bounds as maximum and minimum values. Then it checks for the feasibility of a solution given the median between the upper and lower bounds. If there is a feasible solution, then it sets the upper bound to the median, otherwise, the lower bound is set to the median. The value of $a_{i,j}$ is set to zero if $a_{i,j}$ is less than the median. Next, a new median is computed with new upper and lower bounds. These steps are repeated recursively until the median is null and the minimum of the maximum values is returned. Once the minimum of maximum cost is computed, it selects zeros from each row and column such that no row and column has a zero element selected twice or more. Finally, the algorithm applies the matching to the original matrix, thus each mobile sensor is assigned to a unique target so that the maximum distance moved by any sensor can be minimized.

5. PERFORMANCE EVALUATION

5.1 Experimental Set Up

A discrete event network simulation framework OMNeT++ [1] is used to evaluate the performance of our mobility algorithm. We investigated networks of varying side length from $200\,m$ to $700\,m$. A sensing range of $11\,m$ and $22\,m$ are considered. Initially, the network is divided into hexagon cells. The results are averages of 100 to 1000 independent scenarios.

5.2 Evaluation of Mobility Algorithm

First of all, we evaluate the mobility algorithm for coverage performance without consideration of the energy consumption issue under three network areas: $300\,m \times 300\,m$, $400\,m \times 400\,m$, and $600\,m \times 600\,m$. The goal of this experiment is to evaluate the performance of the mobility algorithm under the best proportion of static and mobile sensors proposed in Section 4. In each scenario, we analyze ten instances where the number of mobile sensors varies for a fixed number of static sensors (from 100% to 0% with a reduction of 10%) until the region is fully covered. The results are compared with the analytical result as presented in Figure 3. The simulation results of three networks are very similar. It shows that the mobility algorithm scales very well. Furthermore, the mobility algorithm achieves a better performance than the analytical results, in particular, the networks with less than 45 % of mobile sensors. For the network with a higher fraction of mobile sensors a higher mobile node density is required to guarantee a full coverage. The reason is that only a single movement of each mobile sensor from its initial location to its target is applied in our proposed mobility algorithm. Multiple movements of mobile sensors can increase the coverage performance especially for a network with a higher fraction of mobile sensors. On the other hand, limited mobility is considered in a hybrid network thus our proposed analytical model and mobility algorithm for a hybrid WSN can guarantee a full coverage with high probability.

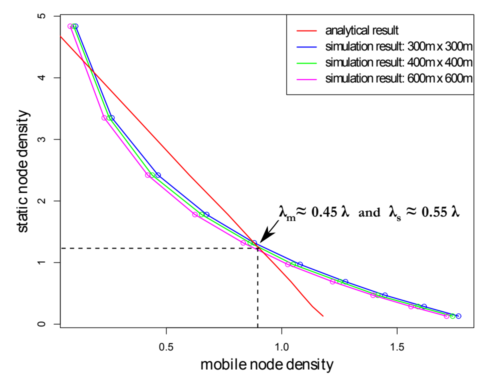


Figure 3. Analytical vs. simulation results of λ_s and λ_m.

To evaluate the proportion of mobile to static sensors as presented in Section 3, we analyze coverage probability under varying the number of static and mobile sensors. Initially, the number of static sensors (n_S = 40, 180, 320) is fixed and mobile sensors are added and mobility algorithm is applied at a time until the network is fully covered. Finally, the required number of mobile nodes is computed. Then we evaluate the proportion of mobile and static sensors that guarantees a full-coverage. According to Table 1, the corresponding analytical results of mobile sensors are 118, 89, and 58 with the node density 1.13, 0.84, and 0.55 respectively. The simulation results of three scenarios are shown in Figure 4 where the required mobile node densities to gain a full-coverage for n_s = 40, 180, 320 are 1.4, 0.7, and 0.5 respectively which are presented as vertical dotted lines. On average, the proportion of mobile to static sensors converges to 4.17 which is better than the analytical result of 4.67 under a network of $200\,m \times 200\,m$ with $11\,m$ sensing range and $f(A) = \sqrt{loglog(A)}$. As we have already discussed in the previous experiment we can see a higher coverage gain for a smaller number of mobile sensors than for a larger number of mobile sensors.

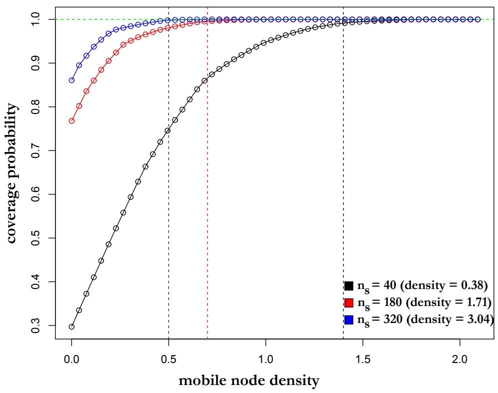


Figure 4. Coverage probability under varying n_s and the corresponding λ_m in a $200\,m \times 200\,m$ network with $11\,m$ sensing range.

We also investigate the performance of the mobility algorithm by comparing it to the optimal deployment. With the help of the proposed mobility strategy the networks gain almost a full coverage of the network. For example, 99.6 % of the network is fully covered in a $200\,m \times 200\,m$ network whereas a $300\,m \times 300\,m$ network is covered at 98.36 %.

Next, we evaluate energy consumption due to node mobility. Three scenarios ($200\,m \times 200\,m$, $500\,m \times 500\,m$, and $700\,m \times 700\,m$) are conducted and the results are shown in Figure 5. Three strategies are analyzed for their energy consumption with respect to the moving distance by mobile sensors as energy consumption is directly proportional to the Euclidean distance. We apply each strategy in the MaxCoverage-MobileSensor-Reposition algorithm where the assignment of mobile sensors to targets is selected randomly in random strategy without optimization for distance movement. As the name describes the min.total strategy is applied in M1 while the min.max strategy is used in the M2 approach. The value of the y-axis is the average or maximum movement per side length of the region. For example, the min.max strategy has 0.4 times the side length of the region $200\,m$ area which is equivalent to a maximum distance of $80\,m$. As we expected, the min.total strategy minimizes the total moving distance whereas the min.max strategy provides the lowest maximum distance. Using the min.total strategy, on average each mobile has to move $42\,m$, $65\,m$, and $72\,m$ for the network with side length $200\,m$, $500\,m$, and $700\,m$, respectively. The min.total strategy reduces the average distance by at least 60 % compared to the random strategy. According to the experimental results, the min.max strategy can minimize the maximum distance by at least 35 % and 60 % compared to the min.total and random strategy, respectively. Both the min.total and the min.max strategy perform better for the larger networks. The choice on which strategy is to be used on a specific application is dependent on the application's desired goal, whether to reduce the maximum distance or the average distance.

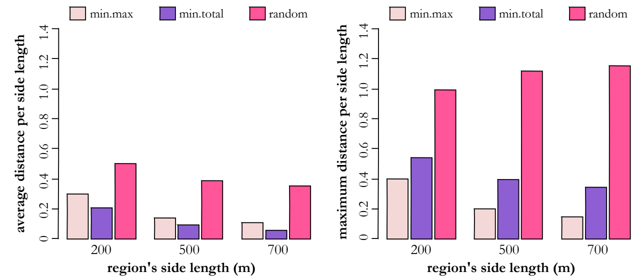


Figure 5. Comparisons of different energy consumption strategies.

6. CONCLUSION

We proposed an analytical model of a hybrid WSN, in particular, the optimal ratio of static and mobile node density. The mobility algorithm with respect to optimal repositioning of mobile sensors and minimizing energy consumption is developed for the mobile portion of sensors. In this paper, we analyzed the performance of the proposed mobility algorithm for one time repositioning of mobile nodes. However, the proposed mobility algorithm can extend to multiple repositionings of mobile sensors whenever topology changes or node failures occur. Finally, the analytical result is validated with simulation results. The results show that the proposed analytical model and mobility algorithm can achieve a full coverage with high probability. Using the optimal repositioning approaches the energy consumption by mobile sensors can reduce from 60 % to 80 % effectively.

7. REFERENCES

[1] Omnet++ manual, http://www.omnetpp.org.

[2] Burkard, R. E., and Çela, E. Linear Assignment Problems and Extensions.

[3] Cao, G., Wang, H., and Laporta, T. A Bidding Protocol for Deploying Mobile Senosrs. In *Proc. IEEE ICNP* (2003).

[4] Derigs, U., and Zimmermann, U. An Augmenting Path Method for Solving Linear Bottleneck Assignment Problems. *Computing Journal 19*, 4 (1978), 285–295.

[5] Fiege, U. A Threshold of ln n for Approximating Set Cover. *ACM Journal 45*, 4 (1998), 634–652.

[6] Kershner, R. The number of Circles Covering a Set. *American Journal of Mathematics* (1939).

[7] Kuhn, H. W., and Yaw, B. The Hungarian Method for the Assignment Problem. *Naval Res. Logist. Quart* (1955), 83–97.

[8] Kumar, S., Lai, T., and Balogh, J. On k-Coverage in a Mostly Sleeping Sensor Network. In *Proc. ACM Mobicom* (2004), pp. 144–158.

[9] Munir, S.-A., Ren, B., Jiao, W., and Wang, B. Mobile Wireless Sensor Network: Architecture and Enabling Technologies for Ubiquitous Computing . In *Proc. AINAW* (2007), pp. 113–120.

[10] Poe, W. Y., and Schmitt, J. B. Node Deployment in Large Wireless Sensor Networks: Coverage, Energy Consumption, and Worst-Case Delay. In *Proc. 5th Asian Internet Engineering Conference (AINTEC)* (2009), pp. 77–84.

[11] Wang, D., Liu, J., and Zhang, Q. Probabilistic Field Coverage using a Hybrid Network of Static and Mobile Sensors . In *Proc. IEEE International Workshop on Quality of Service* (2007), pp. 56–64.

[12] Wang, W., Srinivasan, V., and Chau, K. C. Coverage in Hybrid Mobile Sensor Networks. *IEEE Mobile Computings 7*, 11 (2008), 1374–1387.

[13] Wang, Y.-C., Peng, W.-C., Chang, M.-H., and Tseng, Y.-C. Exploring Load-Balance to Dispatch Mobile Sensors in Wireless Sensor Networks. In *Proc. ICCCN* (2007), pp. 669–674.

[14] Zhang, H., and Hou, J. C. Is Deterministic Deployment worse than Random Deployment for WSNs. In *Proc. IEEE Infocom* (2006), pp. 1–13.