Lower Bounds for Contention in CSMA/CA-Based Wireless LANs

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Abstract

Wireless LANs operating within unlicensed frequency bands require random access schemes such as CSMA/CA, so that wireless networks from different administrative domains (for example wireless community networks) may co-exist without central coordination, even when they happen to operate on the same radio channel. Yet, it is evident that this lack of coordination leads to an inevitable loss in efficiency due to contention on the MAC layer. The interesting question is, which efficiency may be gained by adding coordination to existing, unrelated wireless networks, for example by self-organization. In this paper, we present a methodology based on a mathematical programming formulation to determine the parameters (assignment of stations to access points, signal strengths and channel assignment of both access points and stations) for a scenario of co-existing CSMA/CA-based wireless networks, such that the contention between these networks is minimized. We demonstrate how it is possible to solve this discrete, non-linear optimization problem exactly for small problems. For larger scenarios, we present a genetic algorithm specifically tuned for finding near-optimal solutions, and compare its results to theoretical lower bounds. Overall, we provide a benchmark on the minimum contention problem for coordination mechanisms in CSMA/CA-based wireless networks.

1 Introduction

1.1 Background and Motivation

Operators of mobile telecommunication networks invest large amounts of money for exclusive licenses of certain radio frequency bands and the infrastructure required for providing their services. Consequently, they are interested in using their radio resources most efficiently and therefore put much effort into the network planning process (i.e. employing highly sophisticated models, optimization tools, in-situ measurements, etc.).

Wireless LANs in contrast, are rarely planned with such diligence. This is partly due to the fact that they are in principle easy to deploy, especially when they start out small and grow in an evolutionary process. While their

CSMA/CA random access scheme allows them to co-exist to a certain extent with other nearby wireless LANs, carrier sensing also results in the problem of contention between co-channel senders.

Considering that wireless LANs only have a very limited number of non-overlapping channels to choose from (up to 4 in 802.11b/g, up to 12 in 802.11a, depending on regulations), it is not easy to avoid contention simply by choosing a different channel. The problem might get even worse, if the spatial density of wireless LANs increases, if more vendors adopt proprietary channel bonding techniques [19] to increase the throughput of their products, or if more products that are not 802.11-friendly use the license exempt ISM and U-NII frequency bands. Furthermore, although radio channels in 802.11a are non-overlapping, receivers of many cheaper wireless LAN adapters cannot cleanly filter out single channels. As a result they experience interference from adjacent channels as well.

Thus if locations and configurations of access points (APs) in a wireless LAN are not properly planned, contention is usually unavoidable. The challenge therefore is to introduce mechansims for coordinating access points and stations (STAs) so that contention can at least be controlled.

We are especially interested in the investigation of scenarios where planning AP locations is simply not possible. One particular scenario, in which our basic motivation lies, is that of wireless community networks (WCNs), in which APs are owned and operated by the users themselves, who would like to donate spare wireless access capacity to the community. In this scenario, it is desirable that wireless LAN "cells" from different owners have a large coverage area, thus it would not be a good solution simply to reduce transmission power as much as possible. Yet on the other hand, there is usually no coordination between the APs of the WCN, leading to co-channel interference and contention.

We cannot yet offer a solution for controlling inter-domain contention. However, in this paper we propose methods for determining the optimal assignment of STAs to APs as well as the transmission power and channel settings for both APs and STAs that result in minimal contention for a given wireless LAN scenario. These methods can be used to analyse the potential for reducing contention by introducing coordination between wireless LANs and can also serve as a benchmark for such coordination mechanisms.

1.2 Related Work

As already mentioned in the previous section, much research has been done for the planning of mobile telecommunication networks. One aspect of planning in this context is the selection of installation sites from a set of available candidate sites (e.g. [12, 5]). This can be combined with the configuration of base stations, e.g. choosing antenna types and orientation, and transmission power [7]. Often, the placement problem has multiple, competing objectives, such as maximizing coverage, maximizing capacity, and minimizing installation cost. Channel assignment is another important planning aspect which has been studied both for fixed and dynamic assignments (e.g. [10, 11, 14]). Different heuristics such as simulated annealing [7], genetic algorithms [5], and tabu search [12] have been used for both of these aspects.

Partially, the work on cellular networks can be applied in the context of wireless LANs as well and vice versa. For example, [8] formulated a coverage planning problem for outdoor wireless LANs, but did not consider any peculiarities of wireless LANs, such as carrier sensing or contention, so that their results can be applied in other radio networks as well.

In contrast to this, [2] investigated the WLAN planning problem accounting for the effect of contention introduced by the CSMA/CA mechanism. They give 0-1 hyperbolic formulations and quadratic formulations for the problem of maximizing overall capacity with and without covering constraints and for maximizing fairness with respect to capacity. They only consider a single-channel scenario, but claim that their proposed formulation can be easily generalized to multiple channels.

[15] formulated a channel assignment problem for CSMA/CA-based networks, considering the cumulative co-channel interference from neighboring APs leading to a busy carrier sense signal. Their objective is to minimize the maximum channel utilization experienced by an AP. The authors then proved this problem to be NP-complete and proposed a heuristic, which they applied to two scenarios with known optimal frequency assignments (hexagonally shaped lattice of cells) and uniform, fixed-power sectorized antennas.

[13] provided an integer linear programming formulation, which determines a placement of APs and a channel assignment that maximizes channel utilization in a single step. However in their formulation, APs within interference range have to always operate on different channels, which makes the problem unsolvable for scenarios with many nearby APs and only few available non-overlapping channels.

Finally, [16] proposed a method for joint AP placement and channel assignment which permits co-channel overlapping and aims at maximizing throughput and fairness among stations.

As we are interested in analyzing already deployed wireless LANs from different domains, we do not consider AP placement and we also do not expect to be able to influence the hardware configuration of APs. Instead, we focus on the dynamically adjustable aspects which affect contention: transmission power, channel selection and assignments of STAs to APs. Our objective is to minimize contention experienced by APs *and* STAs by taking into account both direct contention via CSMA/CA's physical carrier sense as well as the virtual carrier sense of the RTS/CTS extension. Note that as a result of transmission power assignment an AP can be switched off, so that we also have some form of selection from candidate sites, but it is not an objective to keep the number of active sites small. Finally, we do not make any assumptions about the size, shape or overlap of co-channel radio cells, as we expect all kinds of heterogeneity to occur in our scenario under study and typically not the traditional hexagonal lattice.

There has also been some work on radio resource management for wireless LANs which is complementary to our work in that our approach can be used as a benchmark for determining the effectiveness of the proposed schemes in reducing contention inside a domain or between domains:

[6] described an architecture in which intelligent switches control APs within a single administrative domain to provide dynamic channel assignment, dynamic transmit power control and load sharing.

[20] proposed an agent-based radio resource management system in which the APs belonging to the same network cooperate with each other to provide full coverage for present STAs and perform load balancing between them.

[17] suggest the use of a radio resource broker that monitors traffic in the connected wireless LANs of different domains as well as the interference between these domains and then compensates networks with high traffic but much interference from other networks by assigning them more channels and transmission power, which it takes away from other domains.

1.3 Contributions and Paper Structure

In this paper, we do not (yet) address mechanisms and strategies for reducing contention by coordinating independent wireless LANs. Instead we take one step back and explore how much benefit it is possible to achieve by introducing coordination at all. To this end we:

- propose a mathematical program for jointly determining the AP–STA associations as well as the transmission power and channel assignment parameters for all nodes of a CSMA/CA-based wireless LAN scenario that minimizes the amount of contention in the system (Section 2.1),
- present extensions of the basic model with only physical carrier sense to additionally consider RTS/CTS and also for the use of service test points for extended coverage (Sections 2.2+2.3),
- show how to calculate a general lower bound on contention in CSMA/CA networks with and without RTS/ CTS (Section 3.1),
- demonstrate how to transform our model into an equivalent linear model that allows us to solve small problem instances exactly using a linear optimizer (Section 3.2), and
- show how to solve larger problem instances using a genetic algorithm which is specifically tuned to our model (Section 3.3).

Finally, we conclude our paper with a short summary and an outlook.

2 Modeling the Minimal Contention Problem

2.1 Networks with Low Traffic Loads

Before a wireless station using CSMA/CA can start to transmit data, it needs to sense an idle channel for a specified amount of time (Distributed Inter Frame Spacing or DIFS in 802.11). Whether a channel is idle or not is determined by a Clear Channel Assessment (CCA) function of the physical layer. Depending on the implementation and the chosen operation mode, the CCA would for example indicate a busy channel when a certain energy detection threshold is exceeded (CCA Mode 1), when a valid signal from another station is detected (CCA Mode 4), or a combination of both (CCA Mode 5) [21]. In this paper we assume that physical carrier sense is solely based on detection of valid signals from other stations. The reason for this is that the default energy detection threshold is usually much higher than the signal level at which transmissions from a single stations can be detected. Only in the rare case that a station receives simultaneous transmissions from multiple co-channel stations (i.e. when these stations sense an idle channel both with physical and virtual carrier sense) would CCA Mode 1 detect a busy medium when CCA Mode 4 doesn't. Furthermore, the energy detection threshold is usually only adjustable in higher-priced equipment.

The signal strength above which a station is able to detect valid transmissions from other stations is typically much lower than the signal strength required for receiving transmissions at a desired data rate. Thus, a station which is farther away from a sending station than the intended receiver might still be restrained from sending to any other station, even though its transmission might be unproblematic.

As a first step, we will model a scenario with wireless access points and stations that use only simple CSMA/CA. Later we will extend the model for RTS/CTS operation.

Let i denote a wireless node with $i = 1, \ldots, I + K$, where I is the number of access points (APs) in the scenario and K the number of stations (STAs). Nodes shall be ordered such that $i = 1, \ldots, I$ for APs and $i = I + 1, \ldots, I + K$ for STAs. Each node i can transmit with a transmission power $x_i \in \mathbb{R}$ between 0 and a node-specific maximum allowed power s_i. On the way from a sender i to a receiver m, a signal experiences a path loss given by p_{im}[1]. A receiving node requires a minimum signal strength r_m to be able to decode a frame transmitted at the desired data rate correctly. If a node i receives a signal from another node with a power above or equal to l_i, its CCA will report the channel as busy.

APs and their associated STAs form a basic service set (BSS). A BSS can operate on one of J different non-overlapping radio channels, $j = 1, \ldots, J$. y_{ij} is a binary variable indicating whether node i currently uses channel j or not. We further define a binary variable f_{im} indicating whether a node i (which must be a STA) is currently associated to node m (an AP) and a helper variable e^{pc}_{im} which indicates whether node i is a potential contender of node m. With potential contender we mean that node m is close enough to i that it can detect i's carrier if both are operating on the same channel. In summary, our first model takes as input

- s_i: the maximum transmission power of node i
  $s_i \in \mathbb{R},\ i = 1, \ldots, I + K$
- r_i: the minimum reception power requirement of node i
  $r_i \in \mathbb{R},\ i = 1, \ldots, I + K$
- l_i: the minimum signal power for node i to detect the channel as busy
  $l_i \in \mathbb{R},\ i = 1, \ldots, I + K$
- p_{im}: the signal propagation loss from node i to node m
  $p_i \in \mathbb{R},\ i = 1, \ldots, I + K,\ m = 1, \ldots, I + K$

and the following decision variables:

- x_i: the current transmission power of node i,
  $x_i \in \mathbb{R},\ i = 1, \ldots, I + K$

[1]Note that we assume dBm as the unit of signal strength. Due to its logarithmic scale, losses (negative values) in dB are actually added to the transmission power to calculate the received signal strength.

- $y_{ij} = \begin{cases} 1 & \text{iff node } i \text{ is set to channel } j \\ 0 & \text{otherwise} \end{cases}$
  $y_{ij} \in \{0,1\},\ i = 1,\ldots,I+K,\ j = 1,\ldots,J$
- $f_{im} = \begin{cases} 1 & \text{iff AP } i \text{ is responsible for STA } m \\ 0 & \text{otherwise} \end{cases}$
  $f_{im} \in \{0,1\},\ i = 1,\ldots,I,\ m = I+1,\ldots,I+K$
- $e^{pc}_{im} = \begin{cases} 1 & \text{iff node } i \text{ is potential contender of node } m \\ 0 & \text{otherwise} \end{cases}$
  $e^{pc}_{im} \in \{0,1\},\ i = 1,\ldots,I+K,\ m = 1,\ldots,I+K$

A valid solution of our optimization problem needs to satisfy several constraints, which we will discuss in detail.

First of all, each node's transmission power must be between zero and the node-specific maximum:

$$0 \leq x_i \leq s_i, \quad i = 1,\ldots,I+K \tag{1}$$

All STAs have to receive their minimum power requirement from the AP they are associated to:

$$x_i + p_{im} \geq f_{im} r_m, \quad i = 1,\ldots,I,\ m = I+1,\ldots,I+K \tag{2}$$

Likewise, all APs have to receive their minimum power requirement from the STAs in their BSS:

$$x_m + p_{mi} \geq f_{im} r_i, \quad i = 1,\ldots,I,\ m = I+1,\ldots,I+K \tag{3}$$

All STAs are associated to exactly one AP:

$$\sum_{i=1}^{I} f_{im} = 1, \quad m = I+1,\ldots,I+K \tag{4}$$

Each AP and STA uses exactly one channel:

$$\sum_{j=1}^{J} y_{ij} = 1, \quad i = 1,\ldots,I+K \tag{5}$$

All STAs use the channel of the AP which they are associated to:

$$y_{ij} - y_{mj} - (1 - f_{im}) \leq 0, \tag{6}$$

$$i = 1,\ldots,I,\ m = I+1,\ldots,I+K,\ J = 1,\ldots,J$$

Finally, we force e^{p}_{im} to be 1 if nodes i and m are so close to each other, that m detects the channel busy if i currently transmits on the same channel (for $i \neq m$, of course, since nodes cannot contend for access with themselves):

$$x_i + p_{im} \leq l_m + e^{pc}_{im} M_{im}, \quad M_{im} = s_i + p_{im} - l_m \tag{7}$$

$$i = 1,\ldots I+K,\ m = 1,\ldots,I+K \ \wedge\ i \neq m$$

$$e^{pc}_{ii} = 0, \quad i = 1,\ldots,I+K \tag{8}$$

Considering that a node can only contend for access with another node when both are on the same channel, we are able to calculate a_m, the number of nodes contending for access with node m:

$$a_m = \sum_{i=1}^{I+K} e_{im}^{pc} \left(\sum_{j=1}^{J} y_{ij} y_{mj} \right) \tag{9}$$

Our objective is then to minimize the amount of contention experienced by the nodes in the system:

$$\min \sum_{m=1}^{I+K} a_m = \min \sum_{m=1}^{I+K} \sum_{i=1}^{I+K} e_{im}^{pc} \left(\sum_{j=1}^{J} y_{ij} y_{mj} \right) \tag{10}$$

This optimization problem requires $I^2 + K^2 + 3IK + (J+1)$
$(I+K)$decision variables and $I^2 + K^2 + (J+4)\,IK + 2I + 3K$ constraints and is unfortunately of multiplicative form, which makes it still difficult to solve. In section 3.2 we will show how to make this problem solvable by transforming it into an equivalent linear problem.

2.2 Networks with High Traffic Loads

When traffic in the wireless network increases, so does the number of collisions of transmission attempts. In wireless networks with high traffic loads, a mechanism called RTS/
CTS, first proposed as part of the MACA protocol [9], is usually employed to increase utilization.

In CSMA/CA with RTS/CTS, when a node i wants to transmit data to a node m, it first sends a small Request To Send (RTS) frame containing the receiver address and the duration of the transmission including the final ACK. Upon receiving the RTS frame, m reponds with a Clear To Send (CTS) frame, which contains the remaining transmission duration as well. All other nodes (APs and STAs) which can hear either the RTS or the CTS store the time during which the medium is expected to be busy in their local network allocation vector (NAV) timer and then defer access until the transmission between i and m is over. Since the specified procedure of deferring access is similar to the physical carrier sense described in the previous section, this mechanism is called virtual carrier sense.

Activating RTS/CTS has the advantage, that collisions can in general only occur on RTS transmissions. As RTS frames are comparatively small, the collision probability is significantly reduced. Furthermore RTS/CTS solves the hidden terminal problem, where two stations that cannot hear each other try to send data to the same access point simultaneously. As a drawback, more stations experience contention indirectly, as they are within carrier sense distance of a node receiving a transmission.

We are now going to extend the previous model for the case of CSMA/CA networks using RTS/CTS. This is simple as the previous model already accounts for calculating the number of direct contenders for a given node m. There, a direct contender was defined as a node which, when it transmits, causes m to defer transmissions due to a positive physical carrier sense indication, which is equivalent to the effect of the virtual carrier sense after reception of a RTS frame. All we have to do further is to take into account those contenders i, which interfere with m's transmissions by being able to send RTS frames to at least one node k whose CTS answers m can hear. We call i an indirect contender of m, if it is not a direct contender at the same time, so that the sets of direct and indirect contenders for a given node are disjoint. To indicate that a node is *not* potential contender of another node, we need to define a new helper decision variable e_{im}^{npc}:

$$x_i + p_{im} \geq l_m - e_{im}^{npc} M_{im}, \quad M_{im} = l_m - p_{im} \tag{11}$$

$$i = 1, \ldots I + K,\ m = 1, \ldots, I + K \ \wedge\ i \neq m$$

$$e_{ii}^{npc} = 1, \quad i = 1, \ldots, I + K \tag{12}$$

We can now extend $a_m$with the number of indirect contenders, but have to take into consideration that APs only send to STAs but not to other APs and vice versa. Furthermore, an AP that does not have STAs assigned should not be counted as an indirect contender. On the other hand, if it has STAs, it should be counted exactly once, no matter how many STAs are assigned to it. This is why we introduce the step function $\sigma(x)$. Our objective function thus becomes:

$$\min \sum_{m=1}^{I+K} a_m,$$

$$\begin{aligned} a_m &= \sum_{i=1}^{I+K} e_{im}^{pc} \left(\sum_{j=1}^{J} y_{ij} y_{mj} \right) \\ &+ \sum_{i=1}^{I} \sigma \left(\sum_{k=I+1}^{I+K} f_{ik} e_{ik}^{pc} e_{km}^{pc} e_{im}^{npc} \left(\sum_{j=1}^{J} y_{ij} y_{kj} y_{mj} \right) \right) \\ &+ \sum_{k=I+1}^{I+K} \sum_{i=1}^{I} f_{ik} e_{ki}^{pc} e_{im}^{pc} e_{km}^{npc} \left(\sum_{j=1}^{J} y_{ij} y_{kj} y_{mj} \right) \end{aligned} \tag{13}$$

$$\sigma(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}$$

This model extension adds $(I + K)^2$ decision variables and $(I + K)^2$ constraints. Note that e_{ik}^{pc} and e_{ki}^{pc} always have the same value as f_{ik}, since a STA and the AP it is associated to need to be able to hear each other. We can therefore simply omit these variables in the objective function.

2.3 Networks with Specified Coverage Area

To minimize contention in a particular scenario, the optimizer tries to reduce transmission power as much as possible. In the previous models, the only constraint to this is that the minimum signal strength requirements of all APs and STAs has to be met. As a result, an optimization tends to produce configurations in which the radio cell of each AP is only big enough to reach all its associated stations, resulting in coverage holes between cells. In particular in the context of wireless community networks, this is not desirable behaviour.

In order to enable us to study the case of independent APs providing continuous coverage of hot spot areas as well, we adopted the concept of service test points used in planning of mobile telecommunication networks[7]. Service test points (STPs) define locations at which at least one of the APs must provide the specified minimum required signal strength. Besides this, STPs are completely passive, i.e. they do neither transmit nor receive data and therefore do not contribute to contention themselves.

We redefine i to additionally include STPs as "virtual" wireless nodes: $i = 1, \ldots, I + K + N$. Nodes shall be ordered as before with respect to APs and STAs, but nodes $i = I + K + 1, \ldots, I + K + N$ now denote the new STPs. Our parameters r_i and p_{im} need to provide settings for the STPs as well, and we also need additional decision variables for assigning a STP to an AP that shall cover it.

- $f_{im} = \begin{cases} 1 & \text{iff AP } i \text{ is responsible for STA or STP } m \\ 0 & \text{otherwise} \end{cases}$
  $f_{im} \in \{0, 1\}, \; i = 1, \ldots, I, \; m = I + 1, \ldots, I + K + N$

All STAs and STPs have to receive their minimum power requirement from the AP they are associated to:

$$x_i + p_{im} \geq f_{im} r_m, \quad i = 1, \ldots, I, \; m = I + 1, \ldots, I + K + N \tag{14}$$

The other decision variables, constraints, and the objective functions remain the same, which means that this extension requires IN decision variables and IN constraints more than the previous model.

3 Solving the Minimal Contention Problem

3.1 Theoretical Lower Bounds

In this section, we derive theoretical lower bounds on the minimum contention for both low and high traffic scenarios. The bounds are based on optimistic assumptions about possible contention between APs and STAs, i.e. a best-case analysis is performed for a given number of STAs and APs.

The low and high traffic scenario are distinguished from each other by the fact that the high traffic scenario also takes into account indirect contention induced by the RTS/CTS mechanism, besides also accounting for direct contention between nodes that are within each others' radio range. Let us first derive the more general bound for the high traffic scenario before the bound for the low traffic scenario can only be stated as a special case without indirect contention.

As above, let I denote the number of APs and K the number of STAs. We make the following two optimistic assumptions:

1. APs (and their associated STAs) do not contend with APs (and STAs) of other basic service sets.

2. STAs assigned to a given AP do not contend with each other.

The first assumption requires APs (and their associated STAs) to either be spaced far away enough from each other or to use different channels. The second assumption is optimistic in the spacing between STAs that are associated to the same AP.

Let n_i denote the number of STAs associated to AP i. Under these assumptions the overall contention can be calculated as follows:

$$C = \sum_{i=1}^{I}(2n_i + n_i(n_i - 1)) = \sum_{i=1}^{I} n_i^2 + n_i$$

This is due to the fact that an AP is in direct contention with each of its associated STAs and that each STA is in indirect contention with each other STA associated to the same AP. This contention is minimal if the STAs are as uniformly distributed over the APs as possible:

Proposition: C is minimal if $\forall i, j\ n_i + 1 \geq n_j$.

Proof: Assume C is minimal for a given assignment of STAs to APs but $\exists i_0, j_0$ with $n_{i_0} + 1 < n_{j_0}$. That means $\exists k \geq 2$ with $n_{i_0} + k = n_{j_0}$. Hence, with $A = \sum_{i=1, i \neq i_0, i \neq j_0}^{I} n_i^2 + n_i$

$$\begin{aligned}
C &= A + n_{i_0}^2 + (n_{i_0} + k)^2 + n_{i_0} + n_{i_0} + k \\
&= A + 2n_{i_0}^2 + 2n_{i_0}k + k^2 + 2n_{i_0} + k \\
&> A + 2n_{i_0}^2 + 2n_{i_0}k + k^2 - 2(k-1) + 2n_{i_0} + k \\
&= A + (n_{i_0} + 1)^2 + (n_{j_0} - 1)^2 + (n_{i_0} + 1) + (n_{j_0} - 1)
\end{aligned}$$

This contradicts the assumption that C is minimal for this distribution, as it can be improved by reassigning STAs to APs and thus the proposition must be correct. ■

We therefore make the further optimistic assumption, that the APs achieve a perfect load balancing with respect to their assigned STAs (modulo 1) to find a lower bound on contention for a given number of APs and STAs.

For the high traffic scenario that means the lower bound is given by

$$C = K + m(n+1) + (I-m)n + mn(n+1) + (I-m)n(n-1) \tag{15}$$

where $n = K\, div\, I$ is the number of STAs per AP (possibly plus one) and $m = k\, mod\, I$ is the number of APs with one STA more than others.

For the low traffic scenario we obtain as a special case the following lower bound

$$C = K + m(n+1) + (I-m)n \tag{16}$$

Note that these bounds make very optimistic assumptions on the spatial distribution of nodes and assume enough channels to prevent contention between basic service sets. Hence, in some actual scenarios they can be very loose lower bounds.

3.2 Exact Solving by Linear Transformation

The problem presented in Section 2.3 has a polynominal structure, as the terms of the objective function are products of three and more variables. The binary nature of variables allows us to adopt the technique from [3] to derive an equivalent linear model at the cost of additional decision variables and constraints. For every product of binary variables we introduce a new variable and substitute it with a product which is then transformed to a new constraint.

We substitute $e^{dc}_{im} := e^{pc}_{im} y_{ij} y_{mj}$, $e^{icAS}_{im} := f_{ik} e^{pc}_{km} e^{npc}_{im} y_{ij} y_{kj} y_{mj}$, and $e^{icSA}_{im} := f_{ik} e^{pc}_{im} e^{npc}_{km} y_{ij} y_{kj} y_{mj}$ by adding the following variables:

- $e^{dc}_{im} = \begin{cases} 1 & \text{iff node } i \text{ is direct contender of node } m \\ 0 & \text{otherwise} \end{cases}$
  $e^{dc}_{im} \in \{0,1\}\,,\ i = 1, \ldots, I+K,\ m = 1, \ldots, I+K$
- $e^{icAS}_{im} = \begin{cases} 1 & \text{iff AP } i \text{ is indirect contender of node } m \\ 0 & \text{otherwise} \end{cases}$
  $e^{icAS}_{im} \in \{0,1\}\,,\ i = 1, \ldots, I,\ m = 1, \ldots, I+K$
- $e^{icSA}_{im} = \begin{cases} 1 & \text{iff STA } i \text{ is indirect contender of node } m \\ 0 & \text{otherwise} \end{cases}$
  $e^{icSA}_{im} \in \{0,1\}\,,\ i = I+1, \ldots, I+K,\ m = 1, \ldots, I+K$

The products are then added as new constraints:

Force e^{dc}_{im} to be 1 if node i is potential contender of m and both use the same channel

$$e^{pc}_{im} + y_{ij} + y_{mj} - e^{dc}_{im} \leq 2, \tag{17}$$
$$i = 1, \ldots, I+K,\ m = 1, \ldots, I+K,\ j = 1, \ldots, J$$

Force e^{icAS}_{im} to be 1 if AP i sends an RTS to its associated STA k and node m can hear k's CTS, but not the original RTS

$$f_{ik} + e^{pc}_{km} + e^{npc}_{im} + y_{ij} + y_{kj} + y_{mj} - e^{icAS}_{im} \leq 5, \tag{18}$$
$$i = 1, \ldots, I,\ k = I+1, \ldots, I+K,\ m = 1, \ldots, I+K,\ j = 1, \ldots, J$$

Force e^{icSA}_{im} to be 1 if STA i sends RTS to its AP k and node m can hear k's CTS, but not the original RTS:

$$f_{ki} + e^{pc}_{km} + e^{npc}_{im} + y_{ij} + y_{kj} + y_{mj} - e^{icSA}_{im} \leq 5, \tag{19}$$
$$i = I+1, \ldots, I+K,\ k = 1, \ldots, I,\ m = 1, \ldots, I+K,\ j = 1, \ldots, J$$

Finally we obtain our new linear objective function:

$$\min \sum_{m=1}^{I+K} a_m,$$

$$a_m = \sum_{i=1}^{I+K} e^{dc}_{im} + \sum_{i=1}^{I} e^{icAS}_{im} + \sum_{i=I+1}^{I+K} e^{icSA}_{im} \tag{20}$$

This new formulation can now be solved with any mixed integer program solver. For our evaluations, we have used the open-source software lp_solve[1]. During our initial testing we found out that we could vastly improve the time that lp_solve takes to find the optimal solution, by giving it a hint to use all available channels. We did this by adding the following additional constraints:

$$\sum_{i=1}^{I} y_{ij} \geq 1, \quad j = 1, \ldots, J \tag{21}$$

Note that this hint helped lp_solve to more quickly reduce the search space by enabling a better branching, although it might not have the same effect with other solvers that follow a different branch and bound strategy.

3.3 Solving by a Custom Genetic Algorithm Heuristic

As we have only been able to solve small problem instances exactly with lp_solve so far, we decided to implement a genetic algorithm (GA) that is specially tailored to our optimization models and allows us to study large problem instances as well. Our GA repeats the following steps iteratively until the population has converged:

1. Generate a new generation of individuals by recombining randomly chosen pairs of parent individuals.
2. Mutate each gene of an individual with a probability of $p_{mutation}$. Transmission powers $x_i \in \mathbb{R}$ are mutated by adding a random value drawn from a Gaussian distribution with mean 0 and a standard deviation of σ_i to it, where σ_i is adapted during evolution. Radio channels and AP associations are mutated by randomly choosing a new value from the respective set of allowed values.
3. Finally, we use a tournament selection strategy, where randomly chosen pairs of individuals taken from both parent and child generation compete with each other and the fitter individual of each pair (i.e. one with the lower contention) survives until the next round.

Up to now, the algorithm is pretty much standard. However, we have had good experience with equipping our GA with a special crossover operator and a healing strategy.

According to the building-block hypothesis [4], one should arrange the genes on an individual's chromosome in such a way that those genes that are correlated in their influence on an individual's fitness should be placed close to each other, so that it is less likely that the cross-over operator would tear them apart during recombination. We have therefore arranged genes representing a node's transmission power, channel selection and AP assignment on a 2-dimensional plane instead of the traditional 1-dimensional string, and we have done so in such a way that the distance relationships between nodes are preserved on the chromosome. Our crossover operator then chooses a random straight cut through the chromosome plane, recombining the cut-off chromosome fragments of the chosen pair of individuals.

Furthermore, in order to improve the chances of obtaining a large amount of valid solutions within our population, we apply a healing strategy after each iteration. The healing process involves two phases. First, it searches for nodes whose minimum signal strength requirements are not met and adapts the sender's transmitting power to the required value, if it does not exceed the maximum allowed power. If this is not successful, the healing process tries to find a better AP to associate to for all STAs in turn.

In order to test the quality of results produced by our GA against the optimum results provided by the solver, we have generated 6 different scenarios of 4 APs and 5 STAs each. APs have been placed in locations drawn from a bivariate normal distribution around the center of a 1km x 1km simulation area, with the constraints that they are not closer than 20m and not farther than 150m apart from the next AP. The location of each STA was chosen by picking an AP randomly and then placing the STA within a distance of 10% to 90% of the cell's radius from the AP, drawn from a uniform distribution. We then calculated the path losses between each pair of nodes based on the empirical indoor propagation loss model recommended in ITU-R P.1238-2 [18]. The maximum transmission power s_i for each node was set to 20dBm (or 100mW), which is the maximum power allowed for IEEE 802.11b wireless LANs in Europe. We have set l_i, the minimum signal strength to detect a busy medium, and r_i, the minimum signal strength requirement of a node to -84dBm and -82dBm, respectively, as these are typical values for an Orinoco Gold IEEE 802.11b adapter.

In Table 1 we have listed the minimum contention for all 6 scenarios, as calculated by the GA for 2 to 4 available channels, averaged over 5 independent simulation runs each. The table also shows the minimum contention as calculated by the solver and the general lower bound for networks of 4 APs and 5 STAs, based on our results from Section 3.1. As a worst case estimate, we have further listed the average results of 5 runs of a single, randomly generated solution (Monte Carlo (MC)), with one application of the healing process to generate valid solutions. As the results of our experiments show, the theoretical lower bound can be reached in all 6 scenarios if there are 4 available channels. The fact that the lower bound has been reached means that all but one AP have one STA assigned, the other has 2 STAs. Note that this well-balanced case can usually not be reached in larger scenarios. As the number of available channels decreases, it is not possible to avoid contention between basic service sets anymore in some of the scenarios. Note that in most cases, the GA was able to find the optimal solution.

Figure 1 shows an average of the pairwise difference between the results of the GA and the exact solver as well as of the MC and the exact solver, respectively. It also shows the confidence interval based on the 95%-quantile of the t-distribution. The lower curve shows that the GA almost always finds the configuration with minimal contention. A random but valid assignment leads to much higher contention, especially when there are only few channels available.

To demonstrate the performance of the GA, we have also applied it to two larger scenarios. For the first scenario, we created a simulation area of 3km x 3km regularily covered with 144 APs and then added 66 APs randomly. 400 STAs were placed with the same method as before. The second scenario was similarly created with 64+36 APs and 500 STAs within a 2km x 2km area. In this set of experiments, we have used a simple local search heuristic to further improve the solution determined by the GA. The heuristic works by testing each wireless node, whether a small change of transmission power or a single change of channel would yield any improvement compared to the solution found by the GA. If so, the improvement is made and the probing is repeated, until no further improvements can be found. Table 2 shows the minimum contention

		Available Channels			
		1	2	3	4
LB		12	12	12	12
OPT	1	42	20	14	12
	2	28	12	12	12
	3	17	12	12	12
	4	34	14	12	12
	5	19	12	12	12
	6	33	15	12	12
GA	1	42.0	20.0	14.0	12.0
	2	28.4	12.0	12.0	12.0
	3	17.0	12.4	12.0	12.0
	4	34.8	14.0	12.0	12.0
	5	19.0	13.6	12.4	12.0
	6	33.0	15.0	12.4	12.0
MC	1	58.8	40.0	24.2	27.6
	2	44.8	28.6	23.2	26.8
	3	67.0	46.4	24.2	23.6
	4	62.4	29.6	35.8	24.2
	5	62.2	30.4	28.0	23.4
	6	45.4	29.0	23.4	22.2

Table 1: Minimum contention for varying number of available channels in 6 different scenarios of 4 APs and 5 STAs each.

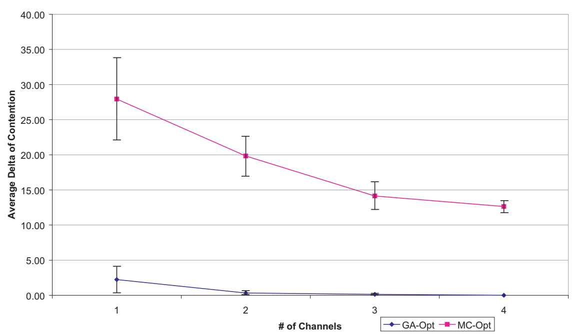

Figure 1: Average of pairwise differences between GA and solver as well as between MC and solver, with 95% confidence interval.

	Scenario			
	200x400		100x500	
LB (no RTS)	800	100%	1000	100%
GA (no RTS)	1202	150%	2199	220%
MC (no RTS)	2255	282%	4872	487%
LB (w/RTS)	1200	100%	3000	100%
GA (w/RTS)	1582	132%	3476	116%
MC (w/RTS)	3026	252%	6822	227%

Table 2: Contention in two large CSMA/CA-based wireless LANs, with and without RTS/CTS.

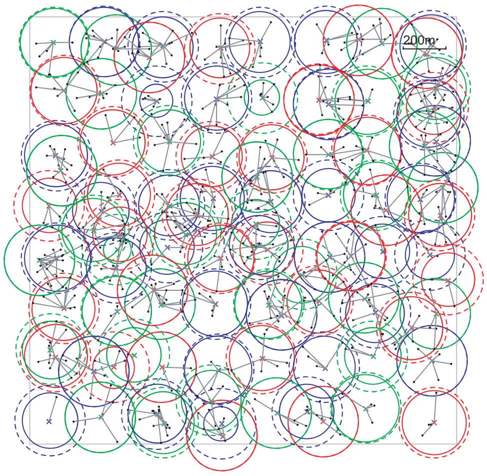

Figure 2: Example of a solved problem.

values found by the GA both as absolute values and as values relative to the theoretical lower bound (LB). For better comparison, we have also provided the contention values of a completely random, but valid configuration (MC). Note that the GA is able to almost halve the contention compared to the random configuration. It is still far from the theoretical lower bound, but as mentioned above, for such sizes it is extremely unlikely that this lower bound is reachable, as it optimistically assumes that perfect load-balancing is possible in the scenario under study. Note also, that minimal contention increases drastically when the RTS/CTS extension is used, as more co-channel nodes from different wireless LANs experience contention due to the virtual carrier sense. Figure 1 shows one of the minimum contention configurations calculated by the GA.

4 Conclusions

Contention in wireless LANs is a result of the CSMA/CA random multiple access scheme. Proper network planning can reduce contention inside a single administrative domain, but is difficult—if not impossible—to do so across administrative domains. As inter-domain contention leads to inefficient use of radio resources, some form of coordination between neighboring wireless LANs should be employed. The objective of our paper was to propose a method to determine a lower bound on contention for a given network scenario, which might improve our understanding of inter-domain contention issues and serve as a benchmark for proposed inter-domain coordination schemes.

In particular we have contributed mathematical optimization models that can be used to jointly determine the optimal transmission power settings and channel assignments for access points *and* stations, as well as the optimal assignments of stations to access points which will result in the least amount of contention in the network. The proposed models cover the case of low traffic intensity, in which only physical carrier sense is used, the case of high traffic intensity, considering the additional contention caused by RTS/CTS frames, and finally the case that a wireless LAN is supposed to provide cell-like coverage by introducing service test points which need to be covered. Further extensions, such as priorization of access points with respect to contention or additional objectives such as power saving for mobile stations can easily be included into our models.

In addition, we have shown how to calculate a general lower bound for contention both in the case of CSMA/CA networks with and without RTS/CTS, we have provided a transformation of our model to make it solvable with linear optimizers for small instances, and we have presented a genetic algorithm which is specially tailored to solve our contention minimization problem, but is likely to be useful in other wireless network optimization problems as well.

Our admittedly preliminary results make us confident that there is much potential for improving inter-domain contention by coordination. They also show that our specially tuned genetic algorithm is able to find near-optimum configurations for the minimal contention problem.

We are currently working on an algorithm to determine a tighter lower bound for contention that makes better use of the peculiarities of the scenario under study and, even more importantly, we are also finalizing our work on a first completely distributed coordination scheme, which we will benchmark using our model.

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